19th INTERNATIONAL CONGRESS ON ACOUSTICS
MADRID, 2-7 SEPTEMBER 2007

AN IMPROVED ENERGETIC APPROACH TO DIFFRACTION BASED ON THE UNCERTAINTY PRINCIPLE

PACS: 43.20.Ei; 43.20.Dk; 43.20.Ei; 43.28.Js; 43.55.Br; 43.55.Ka

Stephenson, Uwe M.[1]; Svensson, U. Peter[2]
[1]HafenCity Universität, Hamburg; Hebebrandstrasse 1, 22297 Hamburg, Germany; post@umstephenson.de
[2]Acoustics Research Centre, Norwegian University of Science and Technology, 7491 Trondheim, Norway; svensson@iet.ntnu.no

ABSTRACT

In room acoustics as well as in noise immission prediction, ray tracing methods or the hybrid method of beam tracing are common. These are energetic methods made for short wavelengths and therefore neglecting diffraction. To introduce diffraction but preserve the advantage of ray tracing, Stephenson has earlier proposed a sound particle diffraction model based on Heisenbergs uncertainty principle. This model has now for the first time been embedded in a full ray tracing program for general set-ups of sources and receivers, and also been transformed into a beam tracing model. This paper compares these new models with the exact wave-theoretical results of Svensson´s secondary edge source model which is based on the exact Biot&Tolstoy solution. Reference cases were the semi-infinite screen as well as two parallel wedges forming a slit. For most cases the agreement is very good (less than 1dB). So, 'Heisenberg' seems to be a useful approach not only for light but also for sound. It is investigated and discussed whether this can be generalized for multiple diffractions. Finally, this new beam diffraction method may also be combined with Quantized Pyramidal Beam Tracing, an efficient algorithm with re-unification for higher order sound diffraction by Stephenson.

INTRODUCTION

In computational room acoustics as well as in noise immission prognosis ('city acoustics') the mirror image source method (MISM) [1], ray tracing (RT) or beam tracing (BT) are used, and these are methods for the optical limiting case of short wavelengths. A version of RT is the sound particle method [2] which, rather than the $1/r^2$-law, uses the more efficient statistical evaluation of the immitted intensities in detectors crossed by the particles. The main deficiency of all these methods is their lack of diffraction simulation. Therefore, the introduction of a diffraction module into ray tracing would be highly desirable, at least as an approximation for short, but not very short wavelengths. The diffraction modeling should fulfill the 'detour law' [3], also for arbitrary diffraction orders and combinations with reflections (but as a pure diffraction module without accounting for flanking walls, reflections are handled by another module). Due to the use of ray tracing as the framework, two basic hypotheses are that diffraction happens only near edges (mainly edges that protrude into a room), and that incoherent (energetic) superposition can be used. However, any combination with the MISM or the sound particle method leads to an explosion of the number of reflection/diffraction-combinations and therefore also the computation time. Best is a straight forward method as RT. Even more convenient is a hybrid method as BT. 'Beams are mirror image sources with built-in visibility limits', so, BT is an efficient version of the MISM. The Geometrical Theory of Diffraction (GTD) [4], or its improvement, the Uniform Theory of Diffraction (UTD) [5], are both high frequency approximations that may in principle be combined with the MISM. Funkhouser utilized a very fast version of BT for auralization in room acoustics [6], even including diffraction in form of the UTD [7]. But still, with higher order reflections and diffractions the computation time explodes.
One of the basic ideas for solving the problem of computation time explosion is: not all combinations and paths of diffracted/ reflected rays or particles are important - only those where particles pass close to edges, where the bending effect on a sound particle is stronger the closer the by-pass-distance. At an edge diffraction event, the particles should be split up into many secondary ones, each carrying a part of the energy of the incident ones. This was the idea of the 'sound particle edge interaction model' (SPEIM) by Stephenson [8], however only for receivers at infinite distance. As this rather heuristical model did not seem satisfying, the first

author succeeded in deriving a sound particle diffraction model analytically from the Fresnel-Kirchhoff theory [9]. This model, on the other hand, failed for more than one edge, so it can not be used as an elementary diffraction module.

With any recursive split-up of rays the number of rays, and hence the computation time, explodes. As long as this crucial problem is not solved, there is no chance for any general ray diffraction model. The basic idea for solving this explosion problem is a re-unification of (similarly running) rays. This is a very difficult problem and only possible if rays are traced in a quasi-parallel and iterative re-distribution process. Also, rays have to be spatially extended, i.e. rather beams, in order to exploit their overlap, to interpolate and to re-unify them. A solution to all these problems is the Quantized Pyramidal Beam Tracing (QPBT) by Stephenson, [10] - a method to efficiently re-unify pyramidal beams.

As intermediate steps on the way to the ultimate QPBT method, this paper applies the 'old' sound particle diffraction model (SPEIM) to more general set-ups, and transforms it to a beam tracing formulation which can then combined with QPBT. So, the SPEIM is then for the first time embedded in a full ray tracing program and also transformed to a more efficient beam tracing.

THE SOUND PARTICLE-EDGE INTERACTION MODEL

The uncertainty principle

The idea that the diffraction effect on a particle is stronger the closer the by-pass-distance is inspired by Heisenbergs Uncertainty-Relation (UR), known from quantum mechanics: $\Delta y \cdot \Delta p_y \approx h$ where Δy is the by-pass distance to the edge, interpreted as the 'uncertainty' in *y*, Δp_y is the impulse uncertainty at the point *y* in space and *h* is Planck`s constant/2π. One may object that acoustics is not happening in atomic scales. But, dividing the UR by *h* (using de Broglie's equation $\Delta p_y = h \cdot \Delta k_y$) yields $\Delta y \cdot \Delta k_y \approx 1$, which is without any atomic constant. This is also a consequence of the Fourier theorem. $\Delta k_y / k$ is then the uncertainty of the direction of the wave vector in the *y*-direction. Analogous equations are valid for the other coordinates. So, it should be possible to utilize the UR to create diffraction algorithms for any kind of particles, photons as well as phonons, hence, it should be valid also for acoustics. This idea has independently and later been successfully utilized in numerical methods for light diffraction to optimize optical systems [11, based on 12]. It should be noted that if the edge (= the *z*-axis) is infinite, then $\Delta z \to \infty$ and there is no reason for any diffraction in the *z*-direction, we do correctly get $\Delta k_z = 0$. This observation is important in the context of the discussion how to generalize the 2D diffraction to 3D: there is no additional effect as edge diffraction happens only in the area perpendicular to the edge, it is basically a 2D effect. According to its nature, the UR must be interpreted statistically. The new diffracted ray direction is never an exact value but obeys a probability distribution of deflection angles, or, equivalently, rays are split up into new ones with partial energies according to that distribution. To allow a modular system, after that 'detour into wave theory' the rays should be superposed energetically. There are two basic concepts in the implementation of this method, the 'Diffraction angle probability density function' (DAPDF) and the 'Edge Diffraction strength' (EDS). For an overview, see fig.1.

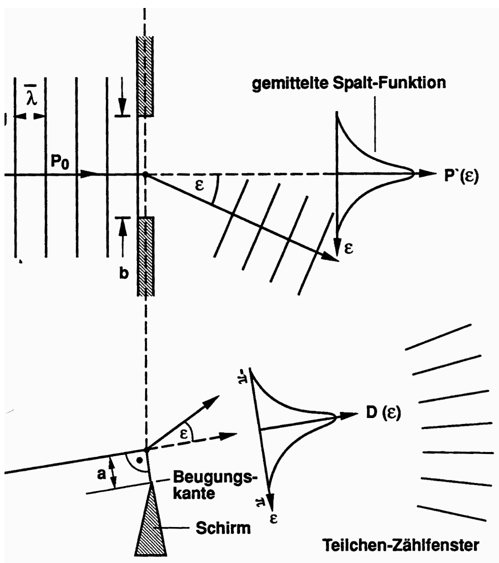


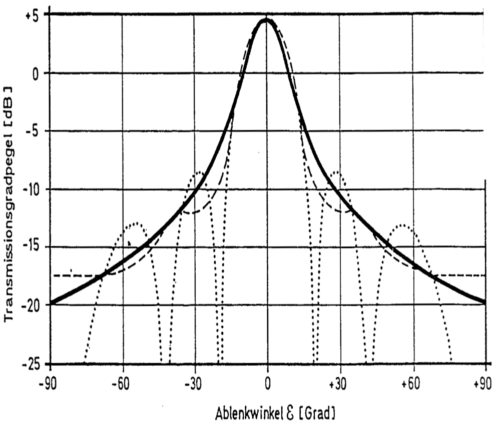
Fig.1: Illustration of the sound particle diffraction model [8]. The moment a particle passes an edge ('Beugungskante') of a screen ('Schirm') at a distance *a* (lower figure) it 'sees' a slit (upper figure). According to the uncertainty relation a certain 'Edge Diffraction Strength' (EDS) causes the particle to be diffracted according to a certain 'Diffraction Angle Probability Density Function' (DAPDF= $D(\varepsilon)$) derived from the diffraction of waves at a slit, see fig. 2. The lower figure also shows an angle window ('Zählfenster') used to count the diffracted particles and to add up their energies to the transmission degree.

The DAPDF

The DAPDF is derived from the well known Fraunhofer diffraction at a slit $\propto \sin^2 v / v^2$, where $v = \pi \cdot b \cdot \varepsilon$. The DAPDF (averaged over a wide frequency band, actually over an octave-band averaged as for 'white light'), is roughly approximated

$$D(v) = D_0 / (1 + 2v^2) \text{ with } v = 2 \cdot b \cdot \varepsilon , \qquad \text{(Eq.1)}$$

where b is the apparent slit width in wavelengths, ε is the deflection angle and D_0 *is a* normalization factor such that the integral over all deflection angles is 1. The D_0-factor must be computed for each edge by-pass since its value depends on b and the angle limits of the wedge. In the following all distances are expressed in wavelengths. (A somewhat improved approach for the DAPDF was used in [8].)



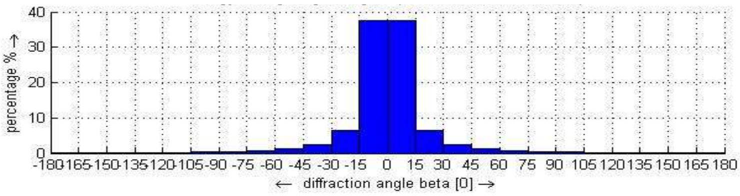


Fig. 2: Left: the derivation of the DAPDF (axes are the deflection angle ('Ablenkwinkel') and the transmission degree in dB) showing the function $\propto \sin^2 v / v^2$ (dashed curve). Right: an energy histogram for a bypass distance of 1/2 and a slit width of 3 (distances rel. to λ). 75% of the incident energy is deflected into the angle range of -15…15°, only 2% into backward directions (<-90°, >+90°).

The EDS

To develop a modular model which is applicable also to several edges that are passed near-by simultaneously, the 'Edge Diffraction Strength' (EDS(*a*)) is introduced such that the EDS of several edges may be added up to a total TEDS, $TEDS = \sum EDS_i$. (Eq.2)

To be used as input for the DAPDF, an 'effective slit width' is $b_{eff} = 1/TEDS$. (Eq.3)

By self-consistency-considerations (a slit should re-produce the energy distribution of itself) it turns out that $EDS(a) = 1/(6 \cdot a)$ (Eq.4)

So, with only one edge, a by-passing particle would 'see' a relative slit-width of b_{eff}=6*a*.

Method of evaluation

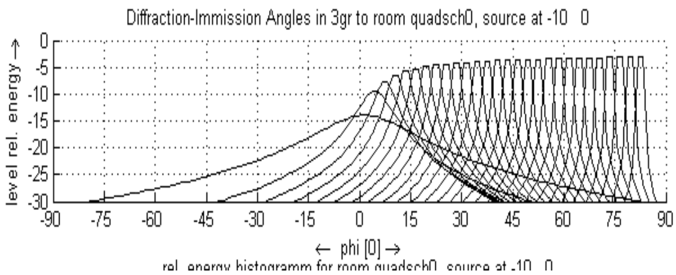


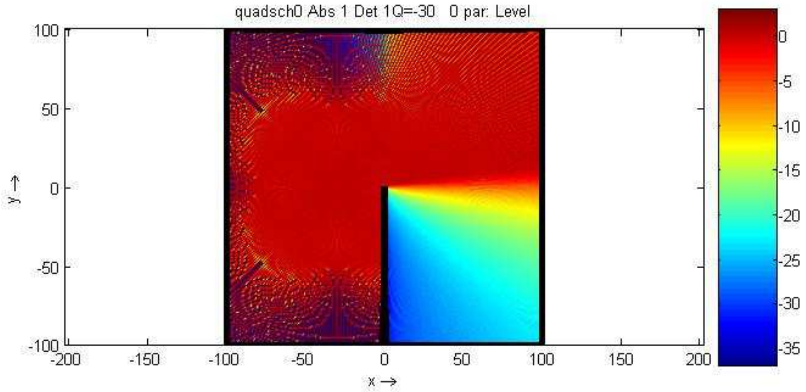
Fig. 3: The superposition of DAPDFs in dB of single particles from a source at -10 λ passing at different distances (see fig. 1) summing up to the screen transmission function (as, e.g., in fig. 7).

The transmission degree is defined as T = intensity with the diffraction of an obstacle rel. to the intensity in free field where 'intensity' in 2D is 'sound power/width' of a detector instead of 'power/surface' but the proportion of T is the same in 3D. This particle diffraction model has now been combined with a full 2D sound particle tracing algorithm. One example of transmission level distribution behind a screen can be seen in fig.4.

RESULTS OF GENERALIZED RAY DIFFRACTION EXPERIMENTS

For a systematic analysis, the 2D ray tracing was evaluated for sources and receivers (detectors of convenient sizes) at finite distances of 1,3,10,30,100 (wavelengths) and 15 angles -84…+84° (in steps of 12°, seen from the edge), appl ied to the semi-infinite screen, in total 375 combinations. This was first compared with the known angle function of the screen [3]. At the first go (without any parameter fitting), the agreement with the reference function (Maekawa) was very good for almost all cases, also for finite distances (standard deviation of <0.5dB, curves similar as in fig.7). Importantly, the reciprocity principle is fulfilled (same levels with a permutation of source and receiver).

Fig. 4: Transmission level distribution for screen diffraction by ray tracing ('room' of 200 λ *200 λ , i.e. 40000 quadratic 1 λ -detectors, 300 primary * 300 diffracted rays, source at x = -30 λ , y = 0; to the right the color legend; bottom right the 'shadow' region with blue colors; 55 immitted particles per detector resp. 0.5dB statistical uncertainty.



Numerically, a decisive quantity is the number of incident particles within a close by-pass distance, a_{min} (which should be about 0.1 λ), and a maximum by-pass distance of a_{max}= 7 λ ; beyond that, direct transmission may be performed. The orientation of the 'diffracting surface' 'above' the screen (dashed lines in fig. 5) has only a weak influence (at +-45° less than 1dB). This is important as in QPBT (the method finally aimed at) a pre-condition for an effective pyramidal beam tracing is a subdivision of the room into convex sub-rooms where on the transparent dividing 'walls' diffraction events at 'inner edges' may be effectively detected.

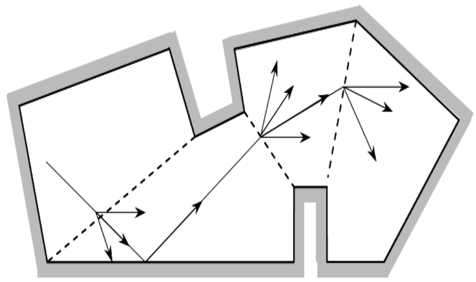


Fig. 5: Subdivision of a room into convex sub-rooms (in 2D): 'transparent' dividing walls are dashed; a ray is scattered/diffracted several times on these 'walls' near edges (only one path is drawn)

FROM RAY TO BEAM DIFFRACTION AND RESULTS

As argued above, the critical re-unification possibility of QPBT requires beam tracing rather than particle tracing to be combined with diffraction. Also the number of secondary diffracted beams could be reduced considerably. In order to reach a certain numerical accuracy particles require a higher number crossing each detector (e.g., about 70 for 0.5dB uncertainty), than beams do, since for mirror image sources, there is no stochastic variation and the $1/r^2$ -distance law may be applied to compute the immitted intensities at the receiver points (in 2D a 1/r-law). To compute the immitted intensities in 2D at one receiver, the following formulae are valid:

For sound particles the typical immission formula [2] | for beams the 1/r-law:

$$T_{BTSP} = \frac{2 \cdot \pi \cdot R}{M_0 \cdot S_d} \cdot \sum_M \sum_{n=1}^{n_0} w_{M,n} \cdot D'(\beta_{M,n}) \qquad T_{BT}' = \frac{2 \cdot \pi \cdot R}{M_0 \cdot \Delta\beta} \cdot \sum_M \frac{D(\beta_M)}{r_{BM}} \qquad \text{(Eq. 5a/b)}$$

where the $0 < D'(\beta_{M,n}) < 1$ are the energy fractions of diffracted rays (integrals of the DAPDF) in the angle range $\Delta\beta$ of the Mth incident and each of the nth diffracted rays, $D(\beta_M)$ is the same for the Mth incident beam –which is also the Mth relevant diffracted beam - , R is the direct distance source-receiver, r_{BM} are the distances bending point -receiver, S_d the detector surface for sound particles and w_{Mn} are the inner crossing distances of particles in detectors. For one receiver, only one loop over all beams (M=1…M_0) is necessary, not a secondary loop over each time an additional number of secondary particles (n=1…n_0).

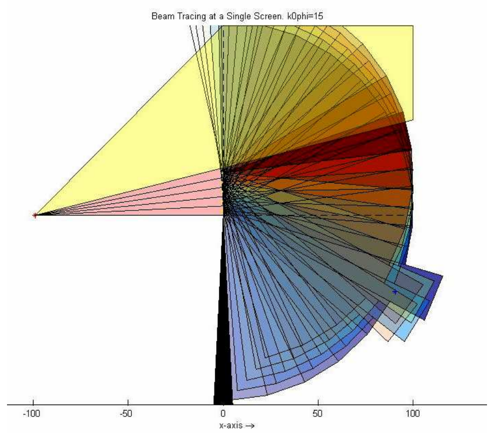
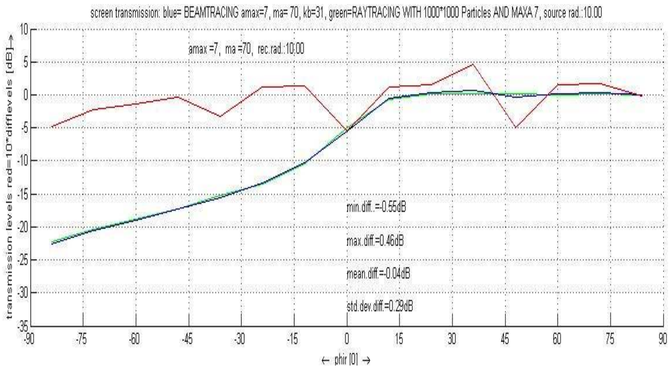


Fig. 6: 2D beam diffraction, specialized for the screen (black wedge in the middle): Typically 10…100 beams ('fans' in 2D) (left, pink) arrive within the decisive by-pass distance range of 0…7 λ (here exaggerated). The direct sound passes above (yellow). To reach all receivers beams are split up into 15 secondary beams (preferably the same number as receiver directions on the rear side). To the right the diffracted beams: the darker the colour the higher the intensity (see fig. 1, resp. the DAPDFs); bottom right the beams relevant for one specific receiver are drawn elongated.

A criterion for the valid by-pass distance of a beam is, as a compromise, the middle ray's distance within the beam. Now, for the 5*5*15 source-receiver position combinations, comparisons were carried out between the beam formulation and the former ray diffraction, see fig. 7.

Fig. 7: Example of comparison between beam diffraction (blue) and ray diffraction (green). The transmission degree in dB is given as function of the receiver angle, to the left the 'shadow' region: only 0.29dB standard deviation. (red curve: deviation* 10) (1000 incident * 1000 diffracted particles , vs. 70 incident * 31 diffracted beams within a_{max}=7 λ ,source and receiver distance: 10 λ , source at y=0).

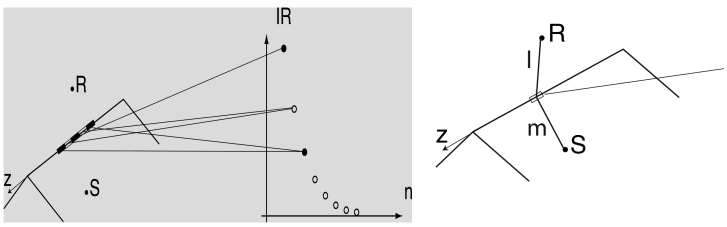


The agreement is very good: the standard deviation of the difference between ray and beam tracing for all 375 combinations is only 0.67dB. Beam tracing is on the order of more than 10 times faster than ray tracing. To exclude any numerical error due to the finite number of beams, a comparison with an 'infinite number' of beams i.e. a (numerical) beam integration was also carried out (over an equ. as 5b). The difference between those results (for the 70*31 beams of fig. 7) was on average only 0.38dB (standard deviation). The direct comparison between beam tracing and the Maekawa screen transmission functions yielded a standard deviation of 0.74dB.

The coherent secondary edge source model as analytical reference model

As preparation for comparisons with more complicated set-ups (more edges) analytical results were needed. Svensson [13] has presented a secondary edge source model in 3D with analytical directivity functions (β , involving the two incident and the two exit angles for each edge source) based on an exact time-domain solution for an infinite rigid wedge [14], [15]. The secondary waves of several edges are superimposed coherently.

Fig 8. Diffraction points on the edge and corresponding samples of the impulse response (left) , *m* and *l* are the distances to and from the edge source (right)



The time-discretized impulse response IR (for sample no. *n* which is proportional to the distance *m*+*l*) can be written as

$$IR(n) = -\frac{1}{4\Theta_w} \int_{z_1}^{z_2} \frac{\beta}{ml} dz \qquad \text{(Eq.6)}$$

where Θ_W is the exterior angle of the wedge, z_1 and z_2 the integration range limits on the edge (=*z*-axis, which may be finite). In contrast to the UTD, the model in [13] is valid also for lower frequencies, but only for hard wedges. Letting edge-sources re-radiate following edges, the method can be recursively applied for higher orders– but (due to the non-spherical secondary waves) with inaccuracies, especially for the slit. The problem of the computation time explosion is not solved either. The IR of the reference model were Fourier transformed and the transfer functions octave band averaged. The standard deviation for all combinations is **only 0.39dB!**

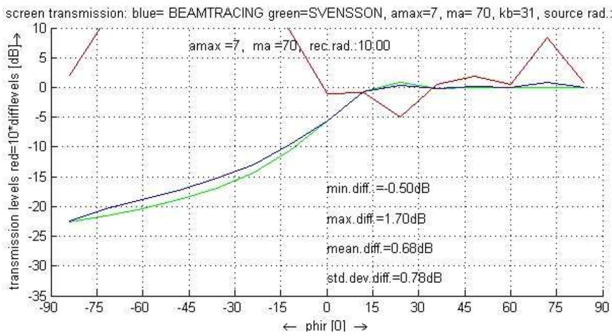


Fig. 9: Example of a comparison between beam tracing (green) and Svensson's reference method (blue) (same legend as for fig.7.)

The reference model permits any wedge inner angle so the influence of that angle was investigated. For smaller inner angles the influence is low, but for the case of 90°, compared with 0°, the differences in the transmission levels are up to 4dB (mean difference are typically 0.4dB). Furthermore, in the reference model, hard flanking walls are assumed (as boundary condition, not the addition of their reflection itself) whereas in the interaction model based on the UR only the position of the edge is relevant, not any flanking walls.

Finally, the diffraction at two edges in parallel forming a slit was investigated (as a self-consistency-test for the SPEIM). So, now the EDS of the two edges were added (Eqs. 2-4).

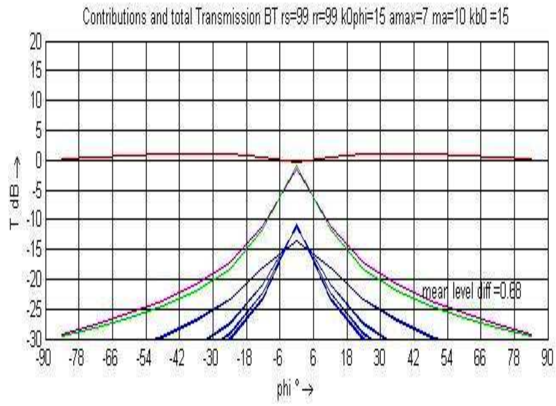


Fig.10: Addition of the DAPDFs of beams crossing a slit of two edges (below); (Source and receiver distance = 99 λ); in the middle (green and violet) the sum i.e. the total transmission as the function of the receiver angle (compared with the free field transmission for the same sound power as incident on the slit); above the deviation curve).

The reference function was the DAPDF of the respective slit width itself. The result was again a very good agreement – at least for far sources and receivers. (For nearer distances, i.e. non-parallel incidence, the agreement can not be good, as the classical slit diffraction function is not valid). In near future, results of a generalization to higher order diffraction will be presented.

CONCLUSIONS AND OUTLOOK

The agreements were in all cases very good.. Consequently, it seems like Heisenberg's UR may be applied also to acoustics and sound may be handled as particles even with diffraction. In principle, it should not be a problem to extend the presented model to 3D and to multiple diffractions. So, a combination of beam diffraction procedures with QPBT is now possible without explosion of computation time. The application to room and city acoustics comes closer.

References:

[1] Borish, J.; Extension of the image model to arbitrary polyhedra. J. Acoust. Soc. Am. 75, pp. 1827-1836, 1984.

[2] Stephenson, U.; Eine Schallteilchen-Computer-Simulation zur Berechnung der für die Hörsamkeit in Konzertsälen maßgebenden Parameter. ACUSTICA 59, pp. 1-20, 1985.

[3] Maekawa, Z.; Noise reduction by screens. Appl. Ac. 1, pp. 157-173, 1968.

[4] Keller, J.B.; Geometrical Theory of Diffraction. J. Opt. Soc. Am. 52, pp. 116-130, 1962.

[5] Kouyoumjian, R.G., Pathak, P.H.; A Uniform Geometrical Theory of Diffraction for an Edge in a Perfectly Conduction Surface. Proc. of the IEEE 62, pp. 1448-1461, 1974.

[6] Funkhouser, T., Carlbom, I., Elko, G., Pingali, G., Sondhi, M., West, J.; A beam tracing approach to acoustic modeling for interactive virtual environments. In proc. of ACM Computer Graphics, Siggraph 1998.

[7] Tsingos, N., Funkhouser, T., Ngan, A., Carlbom, I.; Modeling acoustics in virtual environments using the uniform theory of diffraction. In Proc.of ACM Computer Graphics, Siggraph 2001

[8] Stephenson, U., Mechel, F.P.; Wie werden Schallteilchen gebeugt? In Fortschritte der Akustik, DAGA 1986, Oldenburg.

[9] Stephenson, U.M.; Ein neuer Ansatz zur Schallteilchen-Beugung; In Fortschritte der Akustik, DAGA 1993, Frankfurt, DPG-GmbH, Bad Honnef, 1993, 235-238.

[10] Stephenson, U.; Quantized Pyramidal Beam Tracing - a new algorithm for room acoustics and noise immission prognosis. ACUSTICA united with acta acustica, 82, pp. 517-525, 1996.

[11] Freniere, E.R., Gregory, G.G., Hassler, R.A.; Edge diffraction in Monte Carlo ray tracing. In Optical Design and Analysis Software. In Proc. of SPIE, vol. 3780, Denver, 1999.

[12] Heinisch,R., Chou,T.; Numerical Experiments in Modeling Diffraction Phenomena. Appl. Optics, 10, pp. 2248-2251, 1971.

[13] Svensson, U. P., Fred, R., Vanderkooy, J.; An analytic secondary source model of edge diffraction impulse responses. J. Acoust. Soc. Am. 106, pp. 2331-2344, 1999.

[14] Biot, M. Tolstoy, I.; Formulation of wave propagation in infinite media by normal coordinates with an application to diffraction. J. Acoust. Soc. Am. 29, pp. 381-391,1957.

[15] Medwin, H., Childs, E., Jebsen, G.; Impulse studies of double diffraction: A discrete Huygens interpretation. J. Acoust. Soc. Am. 72, pp. 1005-1013, 1982.