LOW FREQUENCY LIMITS OF REFLECTOR ARRAYS

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1 INTRODUCTION

An array of panels forming a canopy over the stage in a concert hall is commonly used to provide sound reflections for improved foldback to the orchestra. By choosing proper dimensions of the panels and degree of opening area in between, one may easily adjust the balance between the foldback intensity and the fraction of energy transmitted through to the hall. In addition to the numerous acoustical concerns associated with the design of the system, there is often need to provide for lighting and allowance for air circulation in the hall, and to consider aesthetical concerns.

This paper will concentrate on the problem of sound transparency of plane arrays consisting of elements and openings that are small compared to the wave-length. Even if foldback at very low frequencies may be of little importance in some cases, it is critical to know the lower limit for the useful frequency range. There could be many reasons for looking in to small panel design which may introduce an unwanted high pass filter with too high cut off frequency into the transfer function.

Measurements on scale-models of varying reflector arrays have been carried out in order to study the behavior of the size dependant transfer function, and in particular to identify the cut off frequency. Results together with theoretical discussions are presented in this paper.

Intuitively, the term transparency is associated with holes, apertures, openings of various shapes. In this paper the panel arrays will in many ways be treated as large, perforated panels.

The investigations reported in this paper are largely motivated by preparing the design of the new over stage canopy in Oslo Concert Hall in 2004, for which this author was the acoustic consultant. In this case, being able to predict the low-cut frequency was very important.

2 SOME PREVIOUS INVESTIGATIONS

Rindel[1] (1991) has presented some design principles to reflector arrays for improved ensemble in a concert hall, and discussed the useful frequency ranges in a parameter study that included the effects of varying the size of the panels, the total size of the array, and the panel density. Predictions of frequency response arrays of panels with dimensions 0.9*0.9m2 and 1.8*1.8m^2 based on the Fresnel-Kirchhoff Diffraction Integral (in the following referred to as FK) were carried out for the range down to 100Hz. Rindel's principles were very designer friendly in their elegant simplicity.

From the results one could conclude that many small elements should be preferred to fewer large ones, since the FK predicted that larger elements would give uneven foldback to the receiving area for frequencies higher than a limit F_1 depending on element size and its distance from source and receiver. Further, Rindel reported that the FK would predict a low frequency limit F_g dependant on the group (array) size and its distance from source and receiver. In the recommended range between F_1 and F_g, the frequency response depends on the panel area density μ as 20*lg(μ). Low frequency dependence on element size was not discussed. The limitations of the FK will be discussed below.

Ando[2] (1998) presented calculations on arrays consisting of 35 panels, 4 m^2 each, but different shapes in 3 different arrays – triangular, square and circular. From the results, Ando concluded that the triangular elements resulted in less dips in the transfer function than the two other arrays. The results corresponded well with Rindel's estimations between F_g and F_1 , while above F_1 they corresponded with Ando's calculations only when there was a panel in the geometrical reflection path. This is good news for the designer, because it shows that the useful frequency range could extend well above the F_1 limit that otherwise would mean constraints regarding element size.

He further presented measurement results from the large canopy of the Tanglewood Music Shed that confirms that the frequency response is maintained all the way down to 20Hz for this array with its close to 8m wide triangular elements.

Incidentally, all the above investigations involve element sizes and frequency ranges above the bass-leak limit.

Torres[3] (2000) emphasized the significance of edge diffraction to the reflections from panel arrays. From his scale-model measurement results, he states that smaller panels greatly reduce low-frequency reflection, perhaps even more than intuition might predict. In historical view it is worth mentioning the interest for canopy arrays concentrated in the period of 1961-1965 with the case studies by Johnson et al.[4], Meyer and Kutruff[5], and Beranek[6], obviously motivated by the current interest in the designs of the New York Philharmonic Hall and the Tanglewood Music Shed. One of the earliest systematic studies on reflector arrays was presented by Leonard et al.[7] (1964).

Among other contemporary authors who currently are investigating reflector arrays are Cox and D'Antonio, who are announced to give a paper on canopy arrays at the IOA Copenhagen Conference 2006.

3 THEORY

This section intends to present the conceptual basis for this paper.

3.1 Definitions and abbreviations

For the purpose of this paper the following definitions and abbreviations will be used:

- FK the Fresnel-Kirchhoff Diffraction Integral
- FZ 1. Fresnel zone, air-volume in ellipsoid of revolution around the sound ray, defining a channel for un-attenuated transmission between given source and receiver located in each focal point, depending on the wavelength of interest
- S' cross-section inside FZ, i.e. an air-gap or a reflective surface providing ray-path connection between source and receiver, and that may be an integral of separate fragmented dS', by diffraction principles, e.g. a reflector array
- S_0' critical size of S', i.e. an S' that is just large enough not to attenuate a sound ray
- ε edge density of a cross section, $\varepsilon = \ell/S$, where ℓ is length of the edge surrounding the actual cross-section area S
- σ perforation degree, i.e. the total air-gap area divided by the total array area
- μ surface density, i.e. the total panel area divided by the total array area, $\mu = 1-\sigma$
- k wave number, defined by $k = 2\pi/\lambda = 2\pi \cdot f/c$

3.2 Sound ray transmission

Given a source and a receiver connected by a ray-path via a cross-section S', the transfer function for ray-transmission depends in general on the cross-sectional density S'/S_0' within FZ. This effect may be calculated from Rindel (1986[8] and 1991) in the reflection case.

However, the transfer functions for both reflection and transmission does also depend on the size of the wave number k compared to some critical value k_0 determined by the geometry of S'. This paper will present a possible expression for these low frequency limits related to edge density.

To illustrate how ray attenuation depends on both wave number and the FZ, four categories that apply to the reflection case are given in the matrix below, and illustrated in Figure 1. Cut-off due to finite array size ($f<F_g$) is not treated as a separate case, since it may be considered as attenuation due to reduced surface density S'/S_0' inside FZ like in II and IV. Among other significant matters that are outside the scope of this paper is scattering from an array, level changes due to curved surfaces ($S'/S_0'>0$), and typical high frequency problems such as the case of "no geometrical condition" ($S'/S_0'=0$).

	Conditions for ray-transmission	Attenuation
I	Specular reflections	0dB
II	Ray attenuation due to cross-sectional density inside FZ	$\Delta L_{FZ} = 20 \cdot \lg(S'/S_0')$
III	Ray attenuation due to wave number smaller than critical value $k<k_0$	$\Delta L_k = 20 \cdot \lg(k/k_0)$
IV	Ray attenuation due to both wave number and density inside FZ	$\Delta L_{FZ} + \Delta L_k$

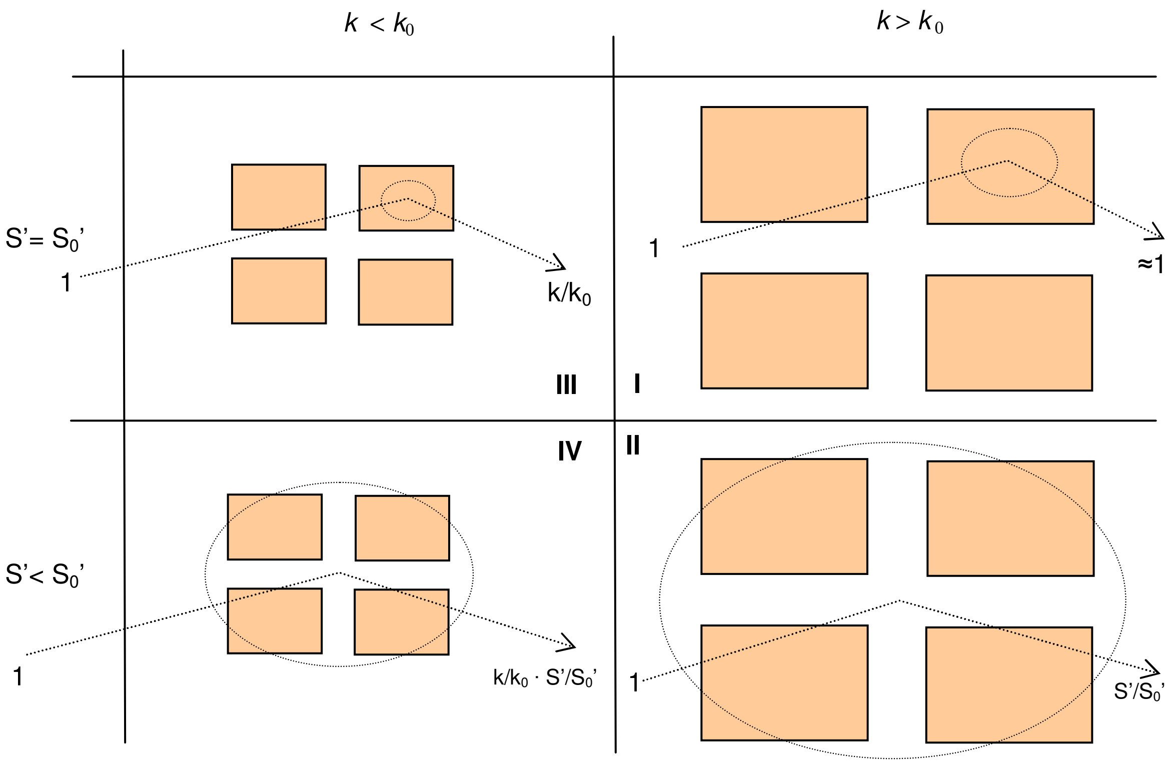


Figure 1: Illustration of the four categories in case of reflection based on relationship between cross-section (colored rectangles), FZ (dotted ellipse), and wave number *k*: **I** and **III** are practically single element reflection cases, treated in literature as specular reflection and scattering, respectively; **II** is a typical $20 \cdot \log(\mu)$ bandpass case, well predicted by FK; **IV** is attenuated due to both FZ and the wave number, only partly predicted by FK. The rays in II and IV apparently re-integrates from fragmented cross-sections. Note: $S'=S_0'$ is valid even if $S>S_0'$, following the definitions of S' and S_0'.

3.3 Cross-sections small compared to a wavelength

3.3.1 Single elements

Sound propagation via air-gaps and surfaces that are small compared to a wavelength has been treated in the literature, as early as 1897 by Rayleigh[9] and 1944 by Bethe[10]. Since then it has been noted by several authors that FK is not valid for predicting such propagation. The reason is that FK is based on the condition that the sound field at the surface of interest is free-field-like with field operator $\nabla = jk$, i.e. out of phase with the sound pressure and having the size of the wave number k.

Though this is a good approximation in many practical cases, it is not likely to be valid close to edges, where one must expect the sound pressure to be associated with gradients due to field divergence. It is therefore more likely to expect ∇ to be real and proportional to the edge-density ε of the cross-section.

From the results in the textbook by Pierce[11] (1981), see 5.3 below, we deduce that FK will underestimate ray-transmission through a circular aperture of radius *a* whenever $ka<4/\pi$ and overestimate ray-reflections from a circular disc of radius a whenever $ka< 3\pi/4$, both with a 6dB per octave tendency. From these observations it is evident that in order to predict low-frequency reflection and transmission, on must establish a separate transfer function

Expressed in terms of edge density and wave number, the low-frequency regions are

$k < \varepsilon\cdot 2/\pi$ for an aperture

$k < \varepsilon\cdot 3\pi/8$ for a disc

This author has looked into the possibility of these expressions being useful approximations for the low-frequency limit of air-gaps and panels general. It seems important to find a general expression for the low-frequency limit for several reasons:

- It has significance in array design
- Given a 6dB per octave behavior below the frequency limit of a reflector array, there will be an abrupt transition between the reflecting mode and the transmission mode.
- A cut-off frequency based on simple geometrical properties, combined with proper 6dB per octave filters, could be implemented in ray-acoustical calculation tools to take attenuation due to sub-wavelength dimensions into account

The effects of cross-sections of sub-wavelength dimensions are in the literature often referred to as diffraction and scattering, the latter also having particular significance in radar technology.

3.3.2 From single element to an array

One can not in general expect that the sound pressure component from one single element will be equal to the sound pressure component from the same element as a member in an array. Some interaction between elements of the array must be expected, so to compute the transfer function of an array may not quite straightforward. Though the dipole behavior prevents one element to impose sound pressure on the rest of the array, one element will indeed impose significant pressure gradients on its neighbours.
It still seems reasonable to expect that an array of elements will show the same 6dB per octave tendencies as for the single elements. For plane waves incident to a non-absorbing infinite array we expect a sound pressure reflection coefficient R=1-T when the sound pressure transmission coefficient is T. This seems a good approximation whenever S_0' is within the array, and reduces the problem to establishing the low-frequency limit.

We shall now investigate the low-frequency limit in the transmission case, considering a plane wave of sound pressure p_i normally incident on an infinite, rigid screen, evenly perforated by circular apertures with radius *a* and with pressure components $p_a(r)$ from each hole transmitted at distance *r* on the receiving side of the screen, see Figure 2. The perforation degree is σ. At a proper distance *r*, and with wavelength λ, the resulting pressure from *n* apertures within S_0' must be $p_t = n \cdot p_a(r)$.

Since plane waves have source distance approaching infinity we must have $S_0' = \lambda\cdot r$, and from Pierce we have the pressure components per hole $p_a(r) = p_i \cdot 2a/(\pi \cdot r)$. The number of apertures inside S_0' is determined from $\sigma = n \cdot \pi a^2/S_0'$, and by combining these equations we find that the sound pressure due to apertures inside the S_0' must be

$p_{S0'} = p_i \cdot 4\sigma/(ka)$

In the appendix, it is shown that the sound pressure component from the S_0' alone is equal to the sound pressure from the whole plane (in practice no screen), we find

$p_t = p_i$

and the sound pressure transmission coefficient

$T = \sigma \cdot 4/(\pi\, ka)$

In terms of the aperture edge density ε_a

the limit for $T = \sigma$ is $k_\sigma = 2\,\varepsilon_a/\pi$ $\quad f_\sigma = 0.10 \cdot \varepsilon_a$

while the limit for $T = 1$ is $k_T = 2\sigma \cdot \varepsilon_a/\pi$ $\quad f_T = 0.10 \cdot \sigma \cdot \varepsilon_a$

We see that the transmitted sound pressure is independent of the receiving distance r, which means that the transmitted sound reintegrates to a plane wave. Since T increases as *ka* gets smaller, T must approach its upper limit at unity as *ka* approaches $\sigma \cdot 4/\pi$, and its lower level σ as *ka* approaches $4/\pi$.

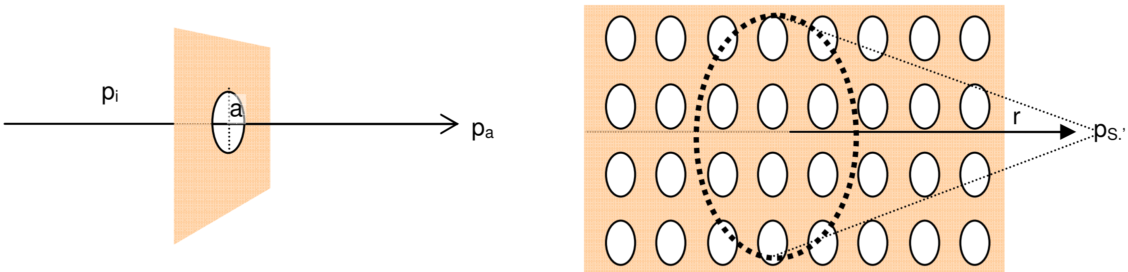


Figure 2: Model for calculating sound transmission through single aperture (left) and through an array of apertures with perforation degree σ, in both cases with plane wave incident on an infinite, hard, rigid screen, and aperture radius *a*. S_0' is contained inside the dotted circle.

A similar deduction can be performed on basis of scattered sound pressure from a single disc, according to Pierce (1981), but assuming $r >> \lambda$ to avoid $\cos\theta$ dependence from the disks:

$p_r = p_i \cdot \mu \cdot ka \cdot 4/(3\pi)$

The reflection coefficient is

$R = \mu \cdot ka \cdot 4/(3\pi)$

As from the aperture array we see that the sound pressure scattered from the array of disks is independent of the receiving distance. Since R must approach μ asymptotically as *ka* approaches $3\pi/4$, we have the cut-off limit for the high pass reflection transfer function R. The cut off limit for reflection, expressed by panel edge density ε_p, wave number, cut-off frequency and speed of sound:

$k_{0,R} = 3\pi/8 \cdot \varepsilon_p$ $\quad f_{0,R} \approx 0.19 \cdot \varepsilon_p \cdot c$

The transfer function T for transmission through the array has a characteristic transition band between the low pass region, and the semi-transparent region, expressed by air-gap edge density ε_a, wave number, cut-off frequency and speed of sound

$$k_{T,trans} = \text{from } \sigma \cdot 2\, \varepsilon_a / \pi \text{ to } 2\, \varepsilon_a / \pi \qquad f_{T,\,trans} \approx \text{from } 0.10 \cdot \sigma \cdot \varepsilon_a \cdot c \text{ to } 0.10 \cdot \varepsilon_a \cdot c$$

3.3.3 Inductance of an array

Cross-sections of sub-wavelength dimensions whether single elements or arrays, are associated with apparent lumped air masses, inertance, curved wave fronts, high pressure gradients along diverging field lines, dipole radiation, reactive effects, accumulator of kinetic energy, induction in analogy to the electric coil. Since $\nabla p=0$ on a rigid panel, it is a perfect gradient reflector. Reflected pressure gradients opposing the incident gradients on the panels will travel sideways, providing delayed response components due to detour paths in the array plane. In addition to the typical low-frequency transparency behavior, it makes the array response significantly more diffuse at high frequencies than would be expected from the visual image.

3.3.4 Discussion on edge density

It has been shown above that the frequency limits in transfer functions does relate to edge densities in the special cases of arrays with circular panels and circular air gaps. This author is currently investigating the possible generality of this relationship in theory and measurements (see below).

3.3.5 Conclusion on sub-wavelength conditions

FK is not valid for estimating ray transmission via cross-sections that are small compared to a wavelength, and the low-frequency limit depends on geometrical properties. The low-frequency limit of an air gap is in general not equal to that of a panel of the same shape and size. Below this limit the transfer functions have 6dB per octave behaviour, with typical high pass characteristics in the reflection case and typical low pass / shelving filter characteristics in the transmission case. To convenience for computations and for the designer of reflector arrays – FK seems to be valid in the useful frequency range, i.e. in the pass-band of the transfer function for the reflection case.

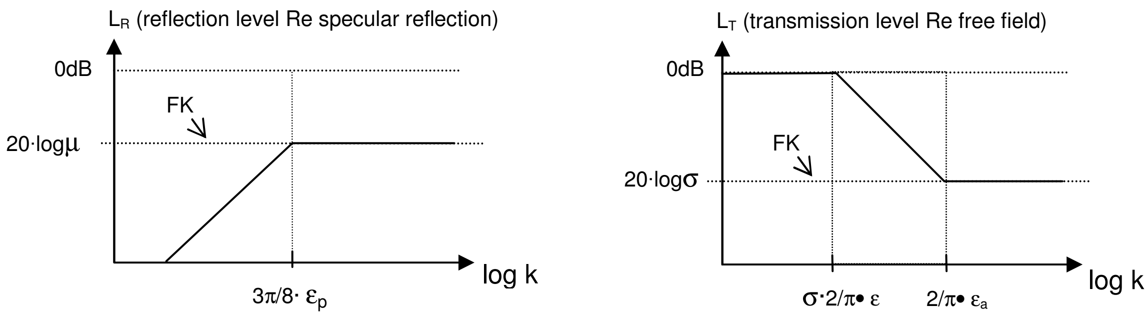


Figure 3 Transfer functions of reflector array as function of the wave number *k* related to the edge density: Basic principles of the expected behaviour due to element size related to wavelengths. The critical frequencies are determined by small geometry properties. Typical filter characteristics are high pass in the reflective case (left), and low pass / shelving in the transmission case. Density relationship is $\mu = 1 - \sigma$.

The basic principles of the expected transfer functions due to sub-wavelength elements in a reflector array are shown in Figure 3 in terms of wave number and edge densities. In Figure 4 we see the same transfer functions with frequency axis. FK predictions are indicated by dotted lines.

Studying the diagrams in Figure 3, considering the case of an infinite, flat panel array with the densities $\mu = \sigma = 0.5$, we have $\varepsilon_p = \varepsilon_a$, keeping in mind the relationship R=1-T and the expectations of a common frequency limit for both reflection and transmission, our theoretical deduction leaves us with two candidates, namely $k_0 = 1.18 \cdot \varepsilon$ and $k_0 = \varepsilon$.

Example: A flat panel array of 1m*1m elements and density μ= 50% has panel edge density ε_p = 4m^{-1} and predicted to have -6dB high pass reflection level with cut-off close to f_0 =160 or 200Hz.

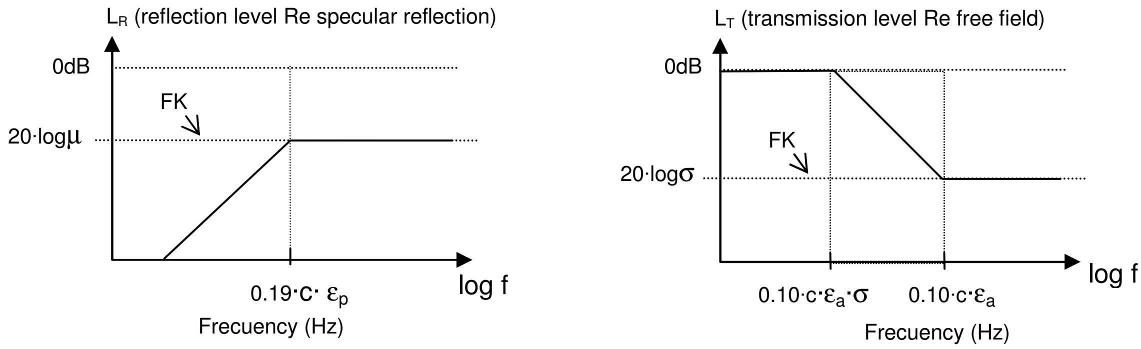


Figure 4: Transfer functions of reflector array; Frequency limits relate to the panel edge density ε_p and air-gap density ε_a. Surface density relationship is $\mu = 1 - \sigma$.

4 MEASUREMENTS

4.1 Measurement description

The aim of the measurements was to investigate the frequency response of reflector arrays, and search for possible significant geometrical parameters.

Scale model measurements of reflection from 6 reflector systems (arrays) with combinations of reflecting surfaces and air gaps, with varying shape, edge densities and surface densities, see Figure 5. The reflecting panels had surface weight 220 g/m^2 and calculated transmission loss with normal incidence >10dB for f > 1kHz, and had negligible thickness related to area. The total array size was 21cm*30cm, and distances from array to source and receiver were both 25cm. Source to receiver distance was 6cm.

The models may look more like air-gap arrays than panel arrays. This was a practical way of providing well-defined edge densities, avoiding the influence from suspension devices, and does not influence the validity of the results of interest in this report, since densities σ and μ does not depend on shape.

Reflected sound pressure was measured and analyzed with winMLS, the frequency responses were related to the 100% density panel (μ = 1), and matched with ideal high pass filter with cut-off frequency as parameter. Correlations were computed to evaluate matching. Correlation between resulting cut-off frequencies and edge densities and other geometrical properties were calculated for evaluation. All results are presented with their true, un-scaled frequencies. In the 50 to 1 scaling examples, the surface weight corresponds to 11kg/m^2.

In order to avoid confusion with cut-off behavior due to finite array size (which is well predicted by FK), all measurements are with the FZ inside the array, and 0dB in all frequency responses is related to the measurement on the reference (un-perforated) object. The frequency range of interest is between Rindels array low-cut F_g and the element limits F_1 and F_2, where FK wrongly predicts constant level.

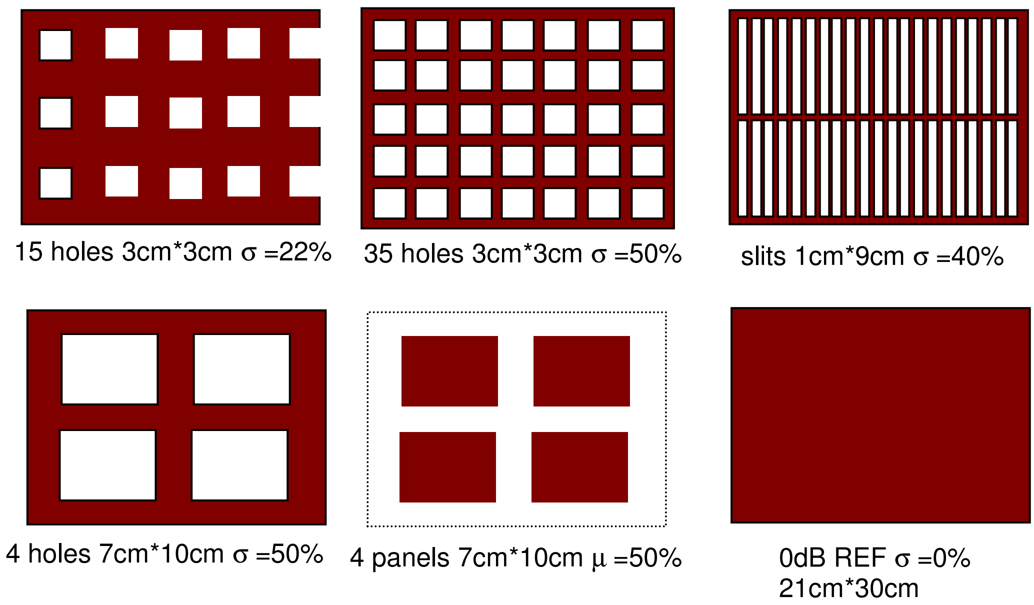


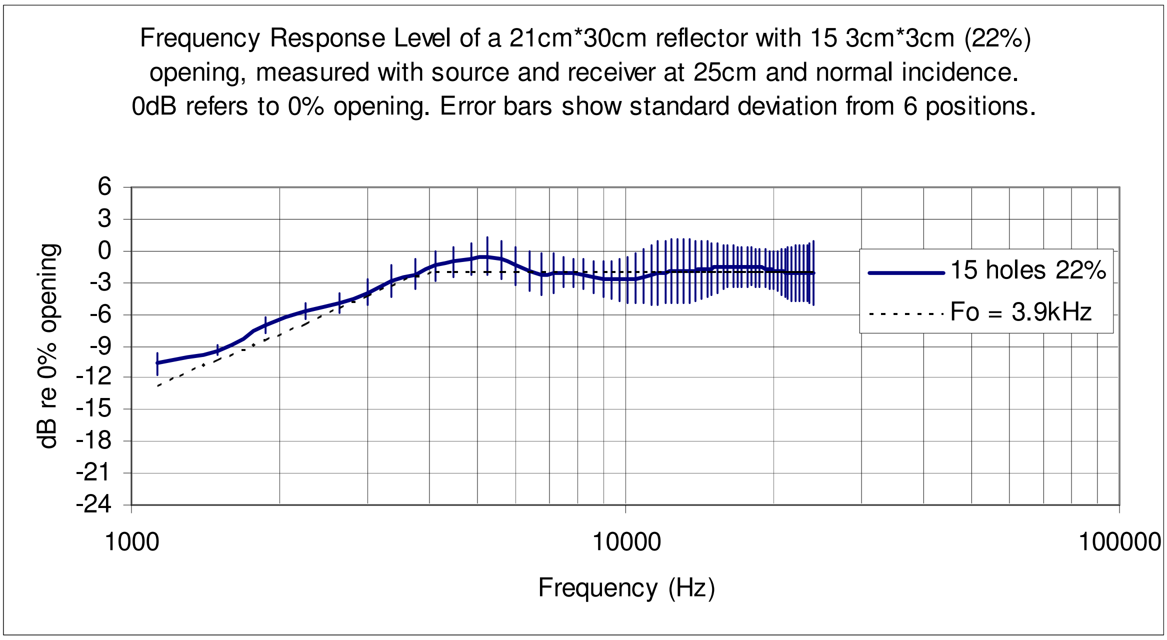
Figure 5: The 6 semi-open array models, including the 0dB reference (un-perforated)

4.2 Results and conclusion on frequency limit

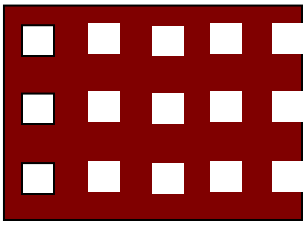
Results from measurements on 6 different objects (5 + reference) are presented graphically with average and standard deviation below.

High pass filters matched well with measurements. Best match cut-off frequencies in terms of wave number correlated 0.99 with panel edge density parameter ε_p, while only 0.56 with air-gap edge densities ε_a. Best match cut-off average from 5 objects resulted in the wave number limit k=1.23·ε_p and frequency limit f = 0.20·c·ε_p with standard deviation ±16%. Thus the measured limit is close to the limit f=0.19·c·ε_p predicted from theory in 3.3.5 above.

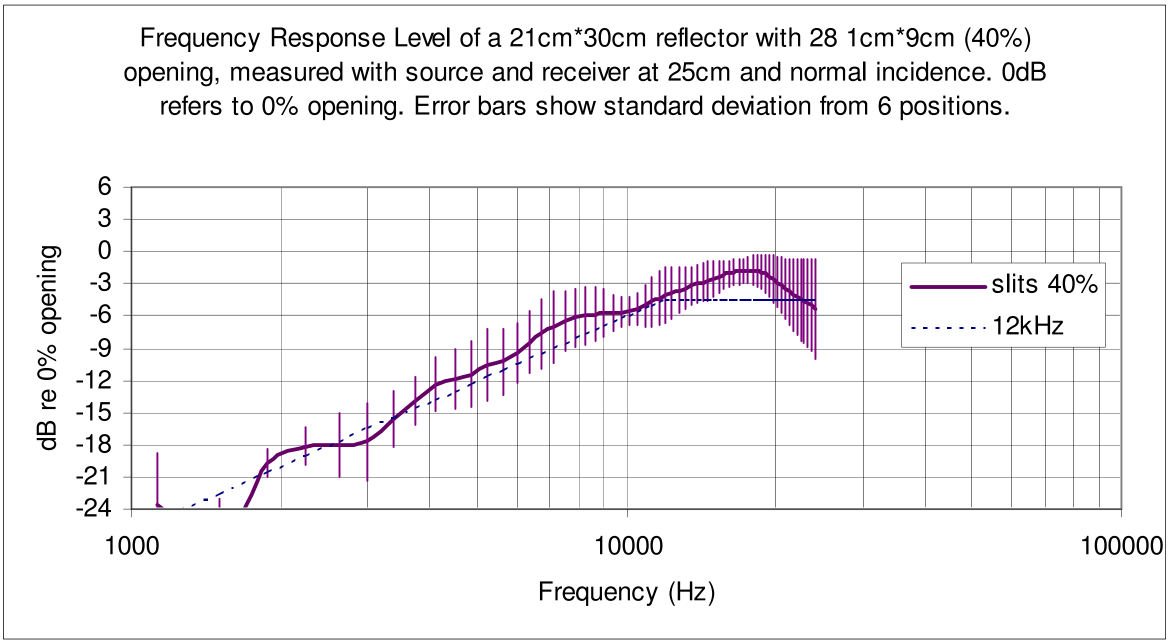
Awaiting further investigations, f = 0.19·c·ε_p , or f = 64·ε_p , is suggested as a rough estimate of the cut-off limit.



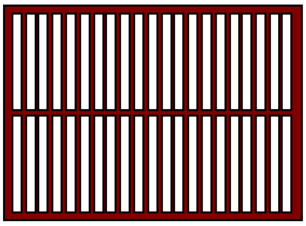
Scale example 50 to 1: Array 10m*15m with 15 holes of 1.5m*1.5m each, cut-off frequency 78Hz



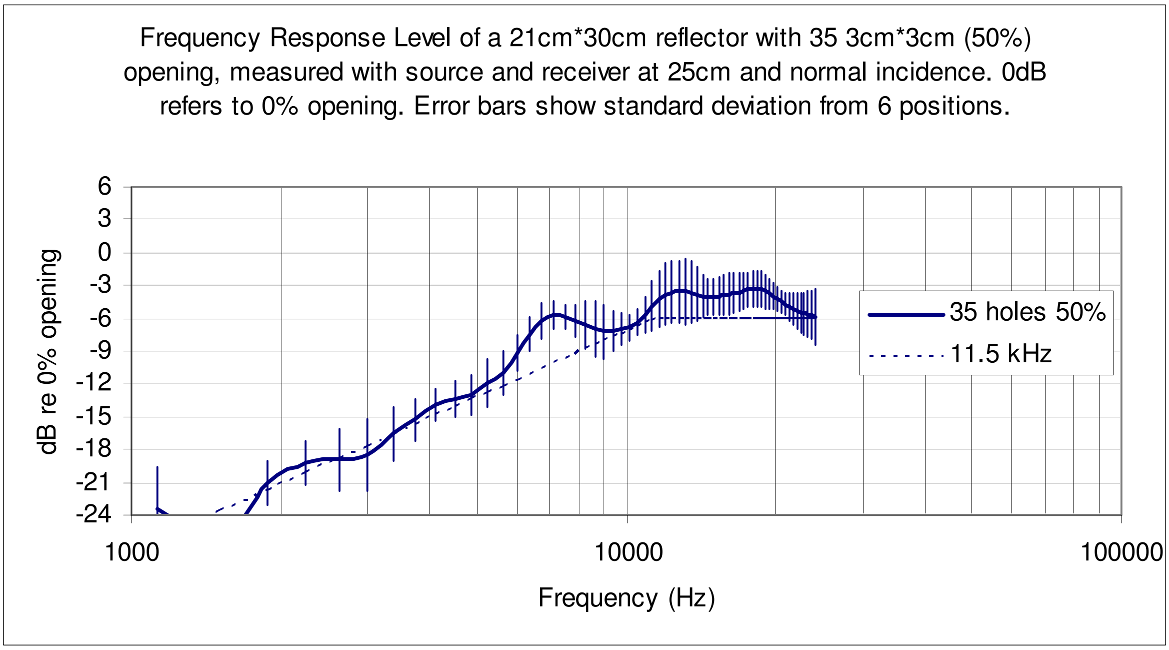
15 holes 3cm*3cm σ =22%



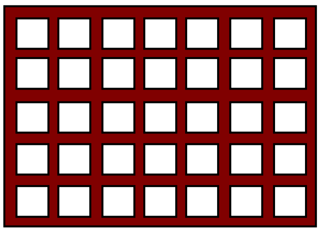
Scale example 50 to 1: Array 10m*15m with 28 slits of 0.5m*4.5m each, cut-off frequency 240Hz



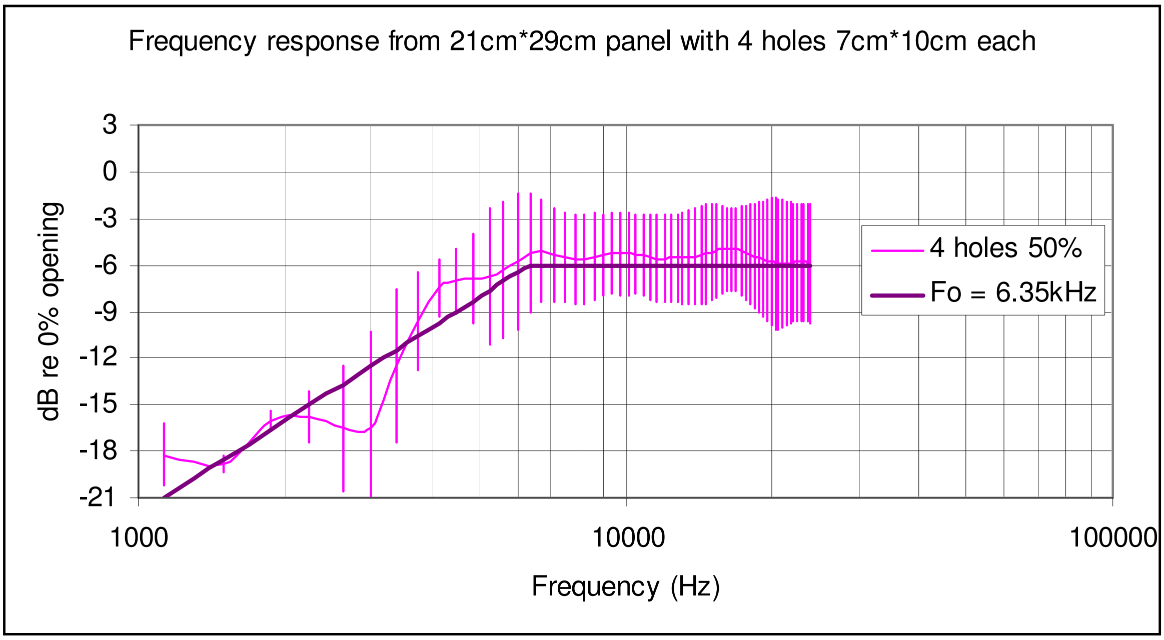
slits 1cm*9cm σ =40%



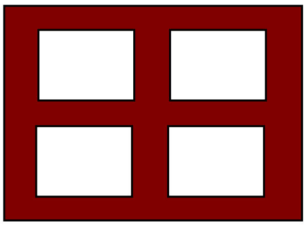
Scale example 50 to 1: Array 10m*15m with 35 holes of 1.5m*1.5m each, cut-off frequency 230Hz



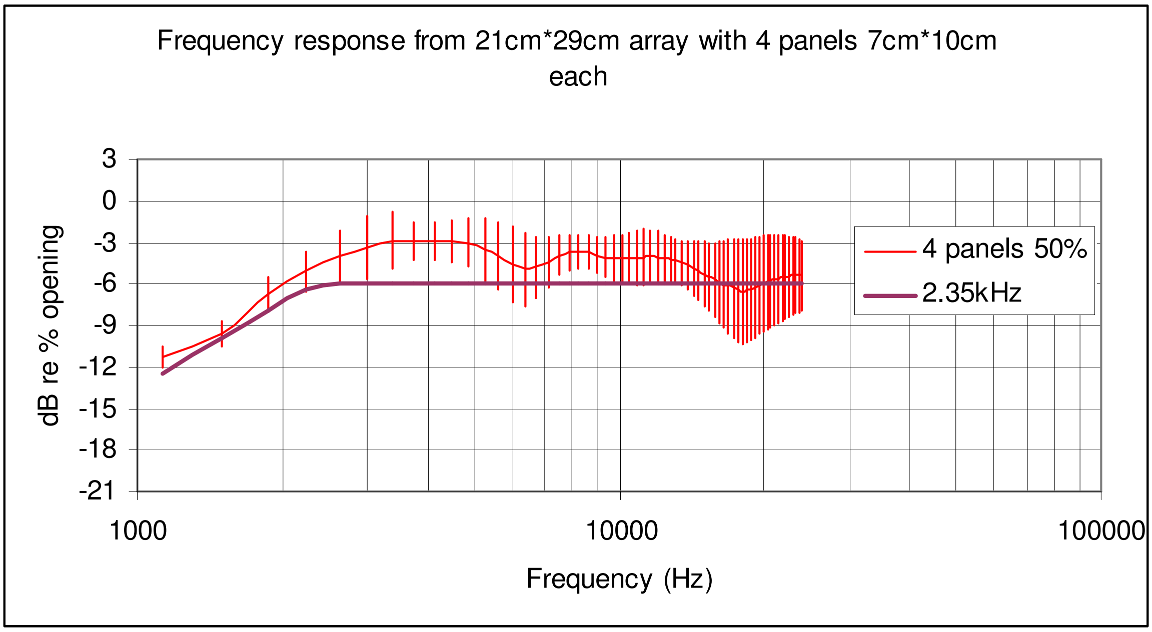
35 holes 3cm*3cm σ =50%



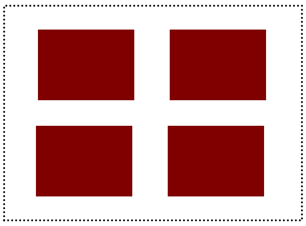
Scale example 50 to 1: Array 10m*15m with 4 holes of 3.5m*5.0m each, cut-off frequency 125Hz



4 holes 7cm*10cm σ =50%



Scale example 50 to 1: Array 10m*15m with 4 panels of 3.5m*5.0m each, cut-off frequency 47Hz



4 panels 7cm*10cm μ =50%

5 APPENDIX

This section is presenting a fuller treatment of the theoretical basis in this document
When a sound wave hits an array of panels, some of the sound energy will be reflected back into the half-space on the source' side of the array, while the rest of the energy will either be transmitted into the half-space on the opposite side of the source, or be absorbed by the array. The reflected energy as well as the transmitted energy will be transferred via the array by combinations of ray-like paths and scattering.

5.1 Rays

Ideal sound rays transmission have some significant features:

- they travel through cross sections larger than the first Fresnel Zone
- cross sections are large compared to a wavelength
- plane wave fronts perpendicular to the rays
- sound pressure gradients by size $k \cdot p$, where k is the wave number and p is the sound pressure
- non-reactive specific impedance equal to the characteristic impedance of the medium, ρc
- they can be predicted by FK

Rays may be attenuated along their path by the size of cross sections.

5.1.1 Ideal ray transmission

Sound energy traveling along straight rays depends on the apparent cross section S' of the surfaces to be reflected by, or the openings to be transmitted through. The ideal cases are the so-called specular reflections from infinite large and rigid surfaces, and the free-field propagation through infinite large openings (in optics often referred to as apertures). In practice such conditions are approximated when the ray path is inside sufficiently large apparent cross sections.

The apparent cross section S' is the cross section "as seen" from source and receiver, and is identical to the actual cross section S whenever the ray path is normal to the cross section. If on the other the hand there is an angle θ between path and normal, the apparent cross section is equal to

$$S' = S \cdot \cos\theta$$

For un-attenuated sound transmission the rays must, all along the path from source to receiver, run through apparent cross sections that are large enough to contain the first Fresnel-Zone, which later will be denoted FZ. More details about the FZ are given in 5.2 below.

The critical minimum size of an apparent cross section for un-attenuated sound transmission can be expressed by

$$S' > S_0' \qquad \text{where } S_0' = \lambda / (1/s + 1/r)$$

where s and r denote the distances along the ray-path from S' to the source and receiver respectively. Readers may find this to correspond with Rindel's frequency limit, above which the sound transmission is un-attenuated by the size of S'.

5.1.2 Ray attenuation due to cross-section smaller than the Fresnel Zone

The general principle for attenuation due to cross-sections $S' < S_0'$

$$\Delta L = 20 \cdot \log(S'/S_0')$$

where S' is the total apparent cross-section inside the FZ, and S_0' the reference apparent cross-section for FZ-related attenuation. For the actual computation, simple formulas are given by Rindel.

5.2 The first Fresnel zone - FZ

The reference apparent cross-section for FZ-related attenuation is introduced by notation S_0' and for the purpose of this document defined as follows: Inside an infinite reflecting plane there is, for a given source and receiver, an apparent cross section that is such that if the surface outside the cross section was removed, the strength of the reflection would be unchanged. This is the reference apparent cross section .
The size of the S_0' varies with distance from both source and receiver such that all sub-sequent S_0's are contained by an ellipsoid with source and receiver in each focal point. In the case of a reflection, one of the focal points will be the image source. The void inside the ellipsoid provides for un-attenuated ray-like transmission along its axis of rotational symmetry. The width or apparent radius of this ellipsoid depends on the wavelength of the sound to be transmitted. Applying common textbook geometry notation, given a source to receiver distance $d=2\cdot c$, the ellipsoid is $2\cdot a$ long and $2\cdot b$ wide mid-way between source and receiver, such that $b^2=a^2-c^2$ and such that $k(2a-2c)=1$, implying $b^2=1/(2k)\cdot(2c+1/(2k))$. Since in most practical cases $2c>>1/(2k)$, we can express the criteria for the width of the ellipsoid surrounding a ray of un-attenuated ray-like transmission by the approximation

$$b^2 \approx d/(2k)$$

Here, b is the apparent radius of S_0' mid-way between a source and a receiver that are distance d apart. By the properties of an ellipsoid the apparent radius y of S_0' can be determined for any distance x from the mid-way position along the ray-path between source and receiver. In most cases we have $kd>>1$, allowing the approximation $a \approx c$, and

$$y^2 \approx b^2\cdot(1 - x^2/c^2)$$

which in terms of x, d and λ can be expressed by

$$y^2 \approx d\cdot(1 - 4x^2/d^2)/(2k)$$

and expressed by the distance s from source to S_0' and the distance r from receiver to S_0', both taken along the ray-path, as

$$\pi y^2 \approx \lambda\cdot s\cdot r/(s+r)$$

Resulting in the size (in m^2) of the reference apparent cross section

$$S_0' \approx \lambda\cdot s\cdot r/(s+r) \qquad \text{for } k(s+r)>>1$$

For an angle θ between ray and the S_0' plane in question, the actual shape of the S_0' will not be circular, but elliptical with the longer axis A so that $y = A\cdot\cos\theta$.

For a circular disc with the area of S at normal incidence for given s and r, the critical wavelength is

$$\lambda = S\cdot(1/s+1/r)$$

which corresponds to the critical frequency of the diffraction filter

$$F_c = c/((1/s+1/r)\cdot S)$$

The expression gives the same result as Rindel's formulas for the critical frequency for a square panel of area $S=A\cdot B$

Readers should keep in mind that, while the S_0' is related to the more commonly used Fresnel Zone, but the latter is bigger, resulting in twice the pressure compared to S_0'.

5.3 Formulas for circular sub-wavelength geometry

Both results are from Pierce (1981): Transmitted sound pressure through a circular aperture of radius *a* in an infinite, rigid, hard screen, from incident plan wave with sound pressure p_i

$$p_a = p_i \cdot 2a/(\pi r)$$

Scattered sound pressure in direction θ from plane wave with sound pressure p_i normally incident on a circular, infinitely thin, rigid, hard disk of radius *a*,

$$p_{sc} = p_i \cdot (ka)^2 \cdot 2a/(3\pi r) \cdot \cos\theta$$

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