Frequency response of Reflector Arrays

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Abstract

This report will investigate the frequency response of reflector arrays. Simplified models for predicting the frequency response and low limit frequencies will be tested with measurements using WinMLS and cardboard array models. The prediction models will be tested with several types of geometry. The report will conclude that the prediction models can be used for normal incidence reflections, but will also reveal some weaknesses.

Contents

1 Introduction

The use of canopies has become more usual in the design of newer concert halls, and in the improvement of old ones. They can have several purposes. They provide foldback to the musician and better communication between all the musicians on stage. They also provide early sound to the audience, which is important for the overall sound quality. The design of canopies varies from one large, single element, to the more typical array of smaller elements. There is however, no key book on how to make a successful canopy, and there are many different issues to consider when designing a canopy [7].

This paper will discuss the frequency response of arrays of small elements. Existing models for predicting the frequency response will be tested with measurements. The theory of these models will be presented briefly. The aim is to find an array design with an even frequency response, ±3dB in the passband. There are two major reasons for wanting a design with even frequency response. The obvious is to have the best possible sound quality, the other is to make further investigations on frequency limits easier and more accurate. This report will be focused on measurements.

2 Previous Work

J.H. Rindel has studied the frequency response of reflector arrays [5]. He used the Fresnel-Kirchhoff approximation to diffraction theory to calculate the frequency response. He studied the parameters element size, array density and array size. He predicted uneven foldback above a frequency dependant on the element size, distance to source and receiver and angle of incidence. He also predicted attenuation below a frequency dependant on the array size, distance to source and receiver and angle of incidence. Between these limits the attenuation is given by the density of the array. He concluded that many small elements should be preferred to fewer larger ones.

T.J Cox and Y.W. Lam [13] discussed Rindel's theories further. They found them to be correct but with some limitations. Incidence angles above 8 degrees where there was a complicated pattern of minima and maxima, rejected the use of a simple high pass filter. R. Torres [14] found with scale model measurements that low frequencies were attenuated more that calculated. M. Skålevik [8] also found an additional low frequency attenuation, which is discussed in the next section.

Others useful sources in the issue of canopy design and diffraction are:

- Y. Ando [1]
- T.J. Cox and P. D'Antonio [11] [12]
- The Tanglewood Shed paper [2]

3 Theory

3.1 Frequency response

M. Skålevik [8] introduced a simplified model of the low frequency response of reflector arrays. This model was based on that the response can be described by two serial high-pass filters. One is caused by attenuation of wavelengths that are large compared to the size of the array elements [6], henceforth the Reflection Filter. The other is caused by attenuation from diffraction [5], henceforth the FK-filter (Fresnel-Kirchhoff). These filter effects can be found on the reflected wave.

The ideal Reflection Filter has no attenuation in its passband, and has a 6 dB loss per octave below its cut-off frequency. Skålevik deducted his empiric formula for the cut-off frequency from scattering theory [3]. He considered a circular disk with radius a in a normal incidence reflection. The limit where the scattered pressure approaches the pressure calculated by the FK-theory, is given by $ka = 3\pi/4$ where k is the wave number. He introduced the parameter $\epsilon = l/S$ which is the edge density where l is the length of the edge and S is the area of an element. For the disk with where $\epsilon = 2/a$ gave the empiric formula for the cut-off frequency.

$$f_c = 64 \cdot \epsilon \tag{1}$$

Scale measurements showed a trend which gave the revised formula for the cut-off frequency [9].

$$f_c = 68 \cdot \epsilon \tag{2}$$

Equation 2 is further tested by measurements in this report.

The FK-filter is described by Rindel's formulas [5][4]. The passband is estimated by $20 \cdot log(\mu)$ where $\mu = S_{el}/S_{ar}$ is given by the element area and the area of the whole array. This filter does also attenuate the reflected wave below a cut-off frequency. This is caused by the fact that the whole array is too small. The array cannot fill the first Fresnel-Zone, so the wave will also partly be transmitted. For normal incidence the cut-off frequency where this happens is given by F_g

$$F_g = \frac{1}{2} c \frac{d'}{S_{ar}} \tag{3}$$

where

$$d' = 2\frac{s \cdot r}{s + r} \tag{4}$$

and s and r is the distance from the source to the reflector and the reflector to the receiver. The same formula gives the high frequency limit for an array of square elements, with the area of one element instead of the whole array. The high frequency limit will not be studied in detail in this report.

$$F_1 = \frac{1}{2}c\frac{d'}{S_{el}} \tag{5}$$

4 Measurements

4.1 Equipment

Equipment used for the measurements:

- Brüel & Kjær sound level calibrator type 4230, 94dB at 1kHz
- 4 Norsonic microphones UC-53N
- 4 Norsonic preamplifiers 1201
- 2 Norsonic frontend microphone amplifiers type 336
- Seas H 615 speaker
- Quad 50E speaker amplifier
- LynxTWO soundcard
- WinMLS 2004
- 105*75cm large mesh of 0.5mm nylon strapped on an aluminium frame
- Panels and elements of 1mm and 2mm thick massive cardboard with even surfaces
- Cables and microphone stands
- Glava mineral wool
- MATLAB R2007a

4.2 Measurement setup

A frame was made of angled aluminium with inner dimensions 105*75cm. A mesh of 0.5mm thick nylon wire was fitted so the mesh consisted mostly of squares with dimensions 4*4cm. The frame was mounted horizontally on two microphone stands. This made it very easy and flexible to spread out different array models. The microphones were mounted at the ends of a cross, with the speaker in the middle as in Figure 1. The four microphones represent four different positions. The microphones and speaker setup was then mounted on a microphone stand and placed 1 meter above the nylon mesh. The distance between the speaker and the microphones were 8 cm, which gave an angle of incidence $\theta < 5°$. This was assumed asymptotic to normal incidence. The equipment was setup in an anechoic room at NTNU to ensure good signal to noise ratio and to eliminate unwanted reflections. The aluminium frame, the microphone stand and the floor grid under the aluminium frame were all covered with mineral wool to reduce unwanted reflections (see Figure 2).

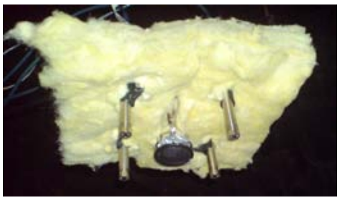


Figure 1: Microphone and speaker setup

Figure 2: Measurement setup in anechoic room

4.3 Validation of measurement setup

4.3.1 Speaker Directivity

The datasheet for the speaker used shows that the speaker has little changes in the frequency response at small angles of incidence. Thorød [10] also found very small changes up to 10 degrees. The directivity was therefore not tested in this project, but assumed omnidirectional for the small angles of incidence used in these measurements.

4.3.2 Unwanted reflections

To make sure that the aluminium frame setup could be used, the frequency response of the reflection from the setup alone was compared with the frequency response of reflection from the setup with a reflector model. Figure 3 shows the frequency response of the reflection from the setup with and without reflector panels. The perforated plots are measurements with a reflector setup. The top plot is one whole panel, and the bottom one is an element array. We can see that the whole panel has an acceptable (e.g 10dB) stronger response than the setup itself, but the element array does not. You can clearly see that around 2.5 kHz the response of the element array is below a limit of 10 dB higher than the response of the measurement setup alone. And around 1.5 kHz the response of the element array actually drops below the response of the measurement setup. This was the case of all the measurements done for validating the setup.

This problem was not solved. This could mean that this setup is not valid for low frequencies. Even though the aluminium frame and other reflecting elements were covered with mineral wool it seems that the setup still contributes too much to make it valid for all frequencies. It was assumed that the aluminium frame would not contribute that much, especially when covered with mineral wool, but it could seem that this assumption was wrong. Due to time limitations on this project, a new setup could not be designed and constructed. A solution was instead introduced in the calculation of the frequency response, presented in section 4.6, and the measurements were carried out with the aluminium frame setup.

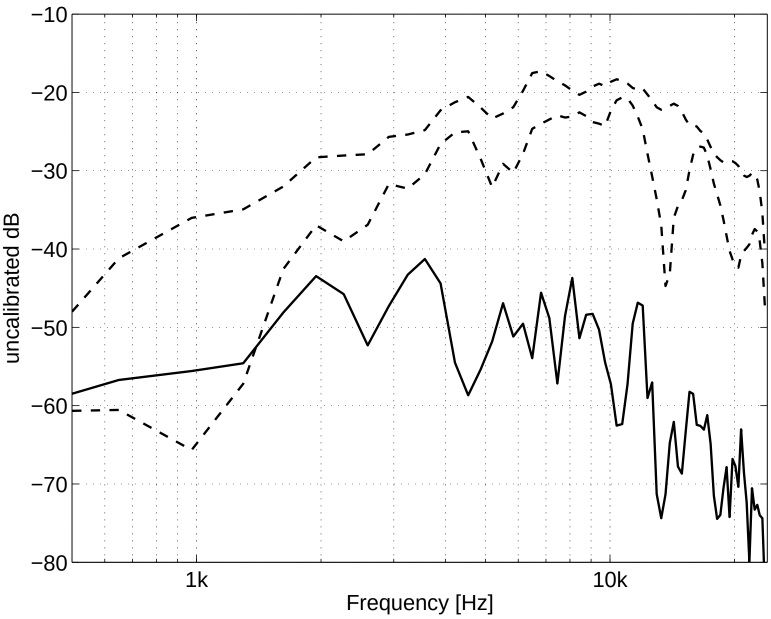


Figure 3: Frequency response of the reflection from the setup with (perforated curve) and without (imperforated curve) reflector panels.

4.3.3 Microphone calibration

The microphones levels recorded varied a lot between the four channels. To compensate for this a microphone calibrator was used. The calibrator was placed on each channel and the signal was recorded, so that a scale factor could be calculated to compensate for the level differences. This was necessary since a mean value between the four channels would be calculated for the results.

4.3.4 Panel materials

Dense cardboard was used as reflector material. One array design used 2mm thick cardboard with an area density of 1kg/m^2. This was found difficult to work with, so the rest of the arrays were made with 1mm thick cardboard with an area density of 0.57kg/m^2. The cardboard had a very smooth surface, so specular reflections were assumed. The reflection coefficient between the air and the cardboard is given by equation 6.

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{6}$$

Where Z_1 is the impedance of the air and Z_2 is the impedance of the cardboard, and are given by equations 7 and 8

$$Z_1 = \rho_0 c \tag{7}$$

$$Z_2 = \rho_0 c + j\omega m' \tag{8}$$

where m' is the area density. If we combine 6, 7 and 8 and allow maximum 1dB loss so that $R = 0.89$, we get the condition that $f > 445\text{Hz}$ when using the 1mm cardboard. This is below the frequency range analysed ($f > 500\text{Hz}$).

$$f > \frac{0.89 \cdot 2\rho_0 c}{2\pi m' \sqrt{1 - 0.89^2}} \tag{9}$$

The cardboard is assumed to reflect all the analysed frequencies adequately.

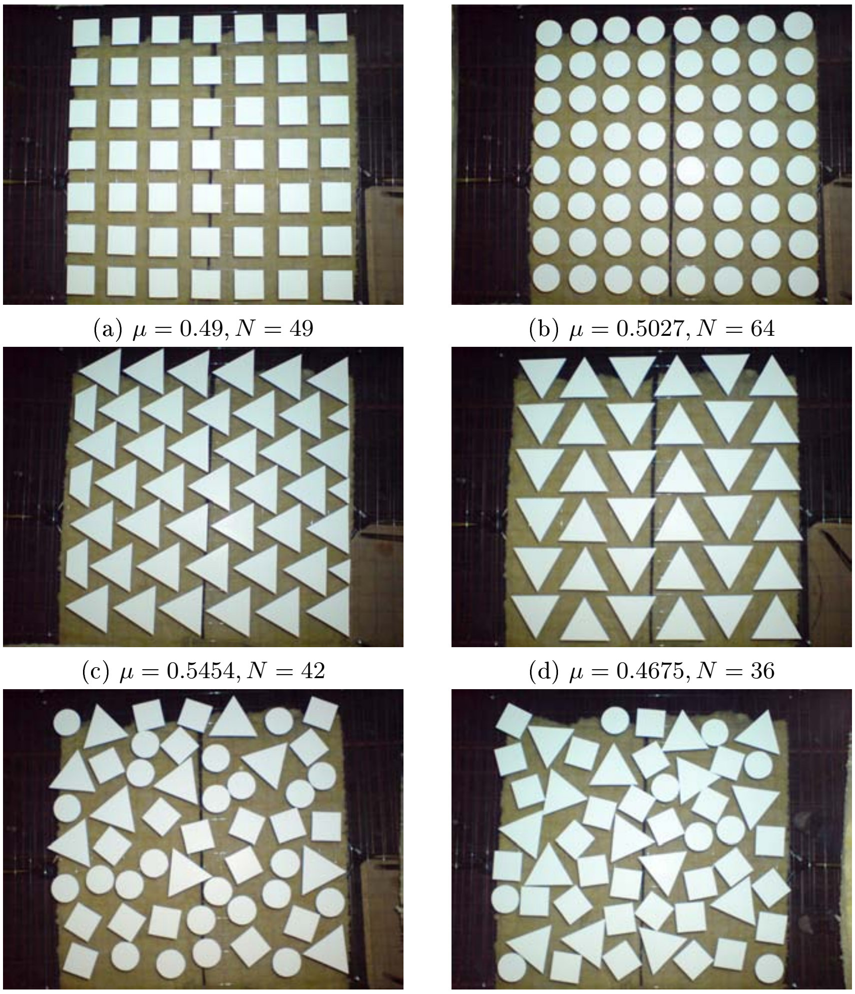
4.4 Array designs

A total of 11 different array designs were measured in this project. All the designs had array dimension $0.7 * 0.7\text{m}^2$ except for the design in figure 6 which had array dimension $1 * 0.7\text{m}^2$. All the designs have the first Fresnel-zone within the array at the lowest frequency, so Rindel's low limit frequency F_g will not be investigated.

All the elements used in the designs in figure 4 had the same value of ϵ which is the predictor used by Skålevik's formula for the low frequency limit. All these designs should according to equation 2 give the same low limit frequency. Measurements on these designs would help to verify if ϵ is a good predictor, or if the low limit frequency could be dependant on other geometrical properties. The circles in figure 4(b) have a radius of 3.5cm, and the triangles and squares used, are sized so the circles can fit inside exactly. Some simple mathematics will show that this will give identical ϵ for all the shapes. The designs in figure 4(e) and 4(f) was included under the idea that a non periodic pattern would give more spread diffraction effects and thus a more even frequency response. The designs in figure 5(a) and 5(b) have the same area as the design of the squares in figure 4(a) but longer edges, which gives a higher value of ϵ and should therefore give a higher low limit frequency according to equation 2.

Two designs with larger elements were included to test Rindel's conclusion that fewer larger elements will give a worse response that more smaller ones. The design in figure 5(c) has element dimensions $17.5 * 17.5\text{cm}^2$, and the one in figure 5(c) has element dimensions $7 * 70\text{cm}^2$.

At last the desing in figure 6 was included as a more complex periodic pattern. The idea was that this pattern could suppress some diffraction effects, and give a more even frequency response.



(a) $\mu = 0.49, N = 49$

(b) $\mu = 0.5027, N = 64$

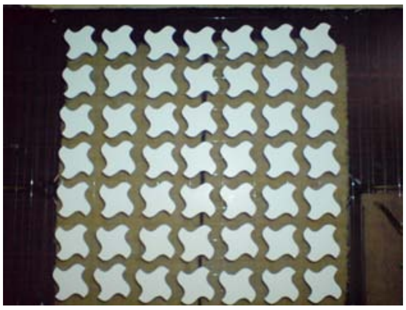
(c) $\mu = 0.5454, N = 42$

(d) $\mu = 0.4675, N = 36$

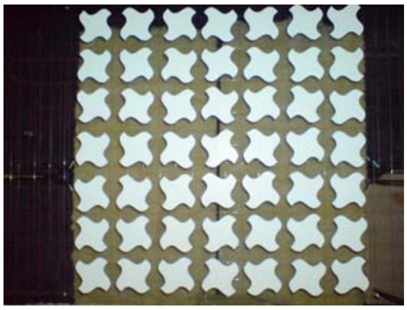
(e) $\mu = 0.5084, N = 53$

(f) $\mu = 0.5425, N = 52$

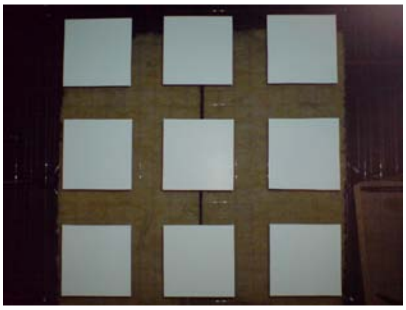
Figure 4: Array designs with the same value for $\epsilon = 57.1429$



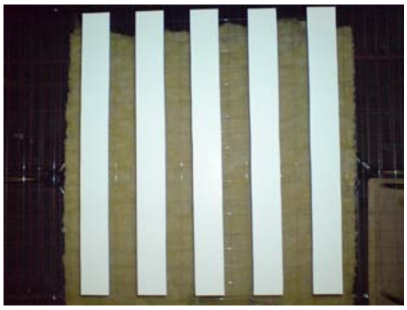
(a) $\mu = 0.49, N = 49, \epsilon = 67.7551$



(b) $\mu = 0.0.49, N = 49, \epsilon = 67.7551$



(c) $\mu = 0.5625, N = 9, \epsilon = 22.8571$



(d) $\mu = 0.5, N = 5, \epsilon = 31.4286$

Figure 5: Array designs with different values of ϵ

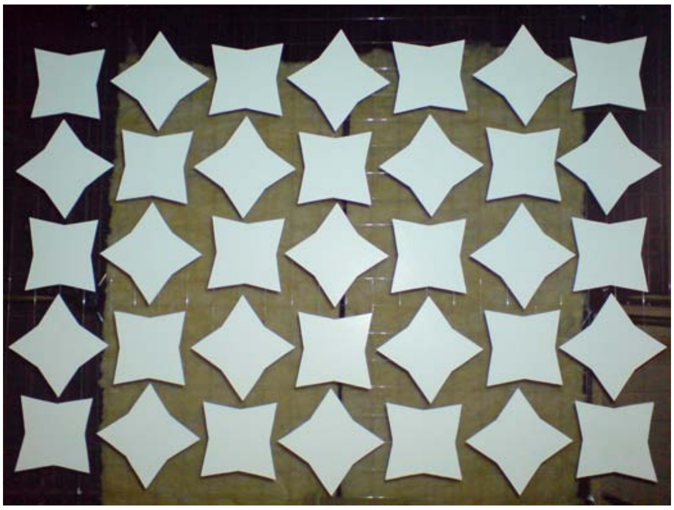


Figure 6: Array of stars $\mu = 0.5050, N = 35, \epsilon = 44.3564$

4.5 Measurement implementation

First a measurement with an array of elements was done, then the elements were removed and a measurement with no reflection panel was done. Last, a measurement of a reference panel was done ($\mu = 100\%$). This procedure was repeated for all the array designs. The measurements were done in this order to minimize the time between the three measurements to have as eaqual conditions as possible. This was necessary in the calculation of the frequency response (see section 4.6). The measurements were done with WinMLS. The four microphones were all measured simultaneously a total of four times for each setup.

4.6 Calculation of frequency response

The measurements from WinMLS were imported as impulse responses in MATLAB. Since the problem from section 4.3.2 could not be solved prior to the measurements, a solution was applied in the post calculation. The impulse response from the measurement setup alone was subtracted from the impulse response from the setup fitted with reflector panels. This improved the low frequency response of the reflection considerably. To ensure that this method was valid, the impulse responses were checked after the impulse response from the setup was subtracted. It was controlled that the direct sound was almost gone, and that the reflection was unchanged. This process was done for all four channels separately.

After the impulse responses were checked, the frequency responses were calculated for both the array of elements and the appurtenant reference panel, for all the channels. All frequency responses were then smoothed to remove very narrow dips in the response less than 1/3 of an octave wide. These dips were assumed to have little significance in a listening situation, although this assumption is not verified. Then the response of the array of elements was divided by the response of the reference panel, so that the reference panel was set to 0 dB. This removed imperfections in the response of the measurement setup, especially the speaker, and also isolated the contributions from the element array. The only effect studied was the effect of the geometry of the elements. The smoothing and the dividing were done for all the channels separated. The smoothed responses were then plotted together with a mean value of all the channels and the theoretical values for the cut-off frequency of the Reflection Filter and passband level given by the FK-filter.

5 Result and discussion

5.1 Array of squares

The frequency response for the array of squares from figure 4(a) is shown in figure 7. Although the mean response is quite smooth in the passband two of the channels have a bit deep dips at 5kHz and 7.5kHz, and above 10kHz there are quite large deviations from the mean value. The dips can be a result of added diffraction effects caused by the periodic pattern. The mean value is perhaps about 1-2dB above the predicted level of the passband. There is a clear attenuation from the Reflection Filter, but the exact cut-off frequency is difficult to determine because of a peak in the response around the predicted cut-off frequency. This peak can also explain that the attenuation has a bit steeper curve at first, but then follows the predicted 6dB behavior.

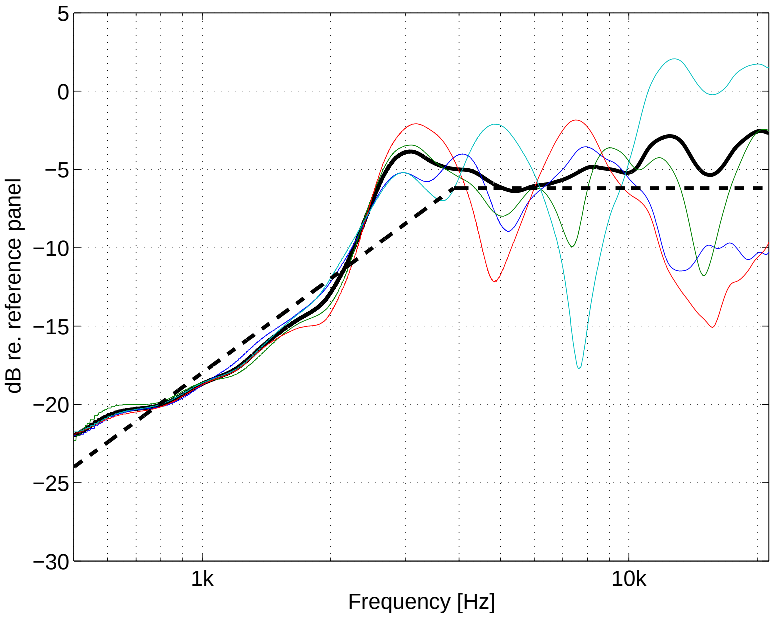


Figure 7: Frequency response of an array of squares as shown in figure 4(a)

5.2 Array of circles

The frequency response of the array of circles, shown in figure 8 also shows a quite smooth mean response, but with large dips and peaks that deviate from the mean value. The passband level is higher than predicted. The same peak can be found around the predicted cut-off frequency, making it difficult to establish it exact , but the effect from the Reflection Filter is prominent.

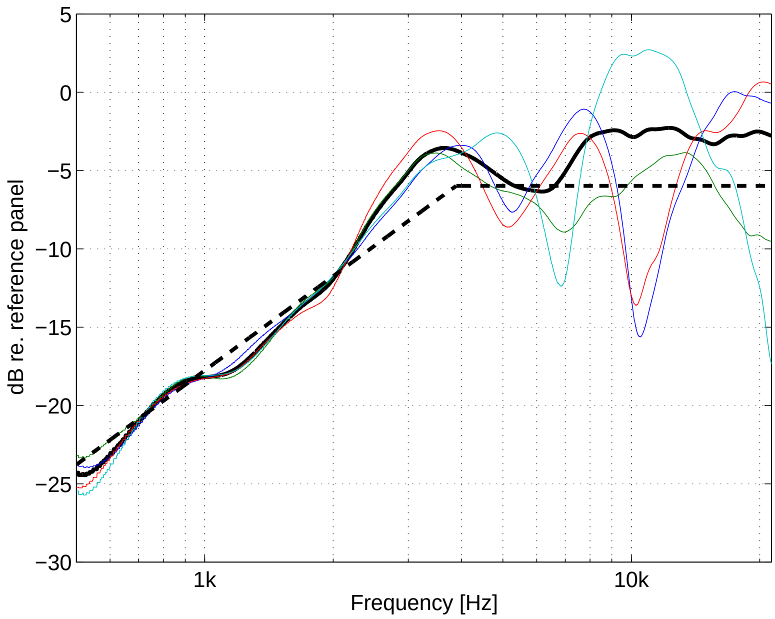
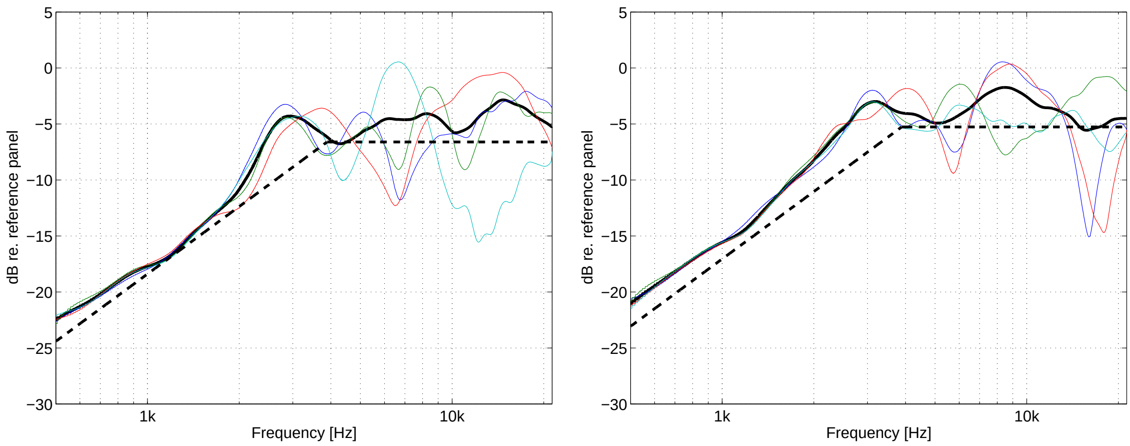


Figure 8: Frequency response of an array of circles as shown in figure 4(b)

5.3 Arrays of triangles

The frequency response of the two triangular array designs are shown in figure 9, shows the same trends as the squares and the circles. Figure 9(b) shows a slightly more even response than figure 9(a) and also has a smoother 6dB behaviour under the cut-off frequency. Both show clear effects from the Reflection filter, and if the passband prediction line is raised the cut-off frequency coincide quite well.



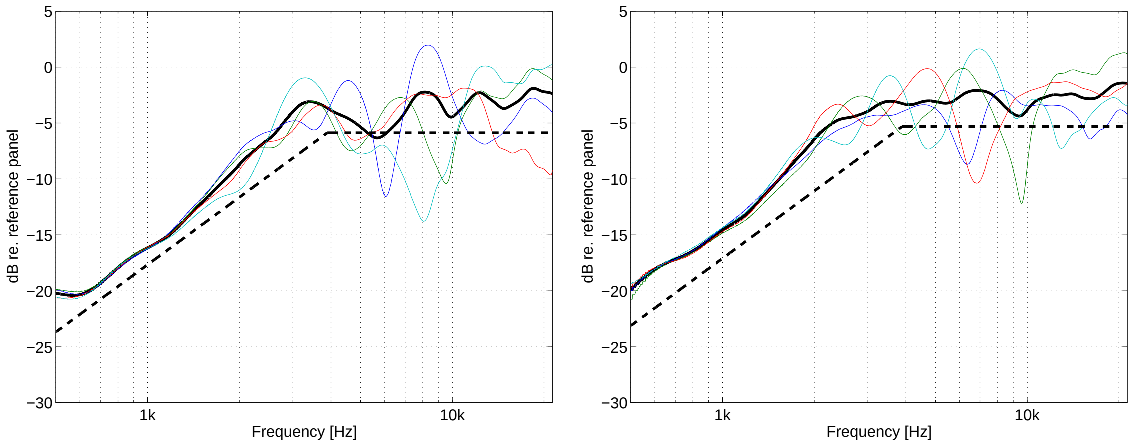
(a) Frequency response of an array of triangles as shown in figure 4(d)

(b) Frequency response of an array of triangles as shown in figure 4(c)

Figure 9: Frequency response of the two triangular desingns

5.4 Arrays of randomly distributed elements

The frequency response of the two randomly distributed designs are shown in figure 10(a) and 10(b). The mean value, especially in figure 10(b) is very smooth, and well within the ±3dB limit. The deviations in each channel are more chaotic than in the structured designs, but they are relatively small. This is also the only design that does not have a peak around the cut-off frequency, and it has a smooth 6dB behaviour. The passband is higher than the predicted value in both cases. If the passband prediction line is raised to the correct level of the mean value, the cut-off frequency prediction will be somewhat accurate.



(a) Frequency response of an array of randomly distributed elements as shown in figure 4(e)

(b) Frequency response of an array of randomly distributed elements as shown in figure 4(f)

Figure 10: Frequency response of the two randomly distributed designs

5.5 Array of stars

The frequency response of the array of stars are shown in figure 11. This array was design with the idea that a more complex periodic pattern could suppress some of the diffraction effects and give a more smooth response. Even though the mean value varies more than the mean values of the more structured designs, the deviations are somewhat smaller. The passband value is found to be higher that predicted, and with a corrected passband prediction line, the cut-off frequency prediction could be correct. This array also have a small peak at the predicted cut-off, complicating the cut-off interpretation.

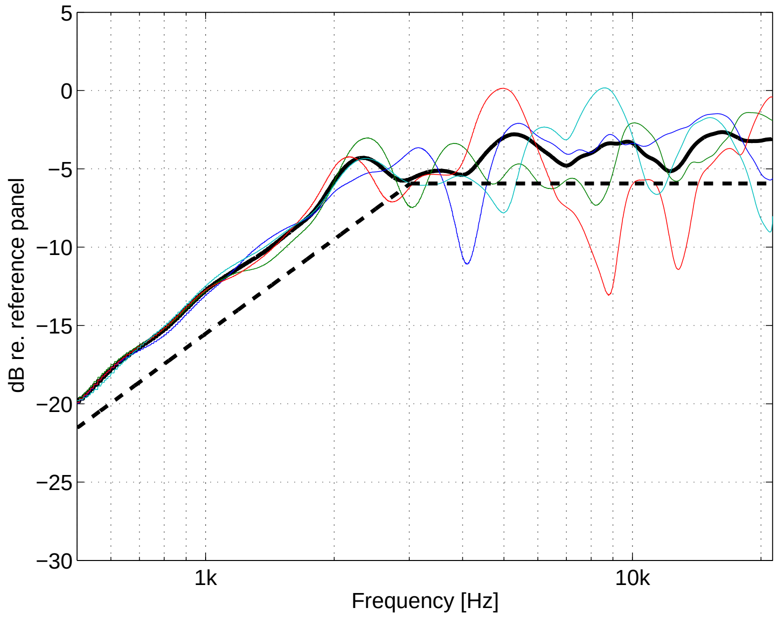
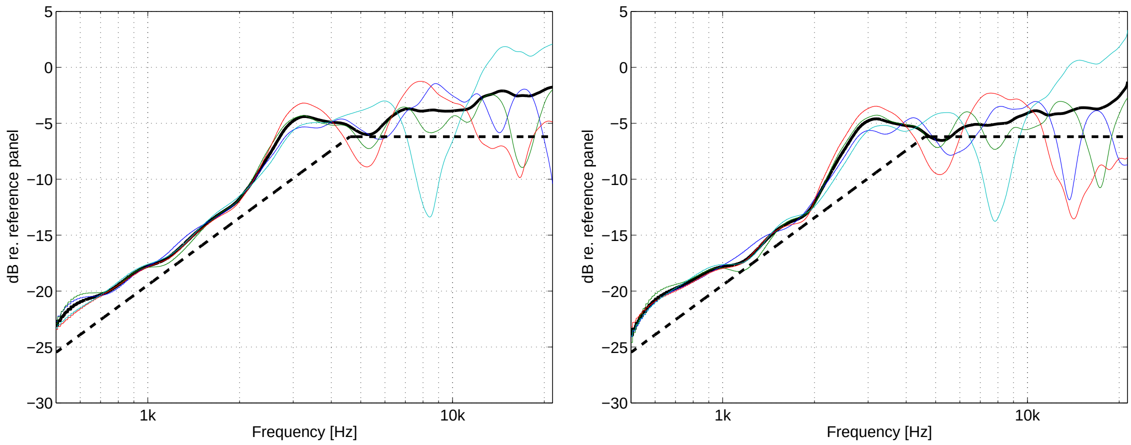


Figure 11: Frequency response of an array of stars as shown in figure 6

5.6 Arrays of squares with wavy edges

The arrays of squares with wavy edges was included cause they have a higher value of ϵ than regular squares, and should accordingly have higher cut-off frequency. The response of two such arrays are given in figure 12. The result does however not support this theory. The theoretical values of the cut-off frequency between squares and squares with wavy edges are close, the ϵ value of the wavy edges is not that much higher, and this might be the reason that no effect of longer edges can be seen. There might, of course be a geometric restriction in the use of ϵ as a predictor for the cut-off frequency, but nothing can be concluded from these results.



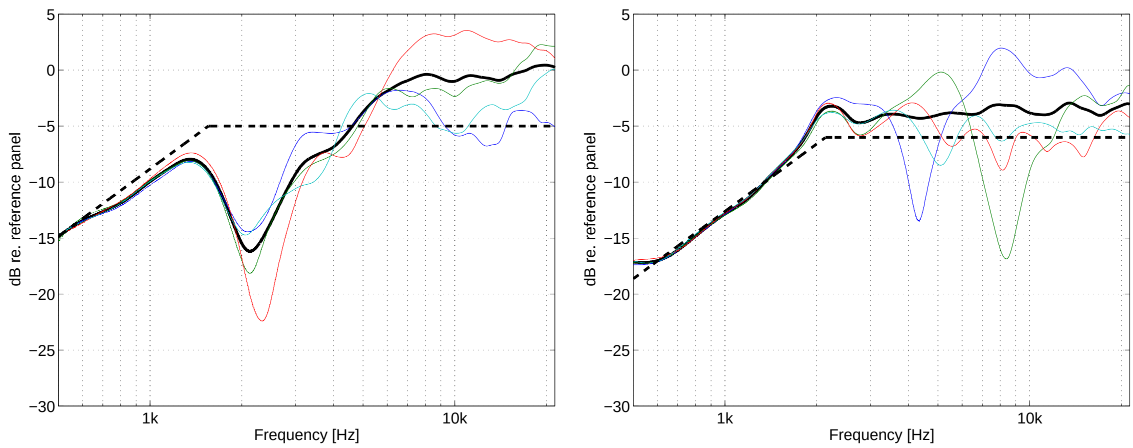
(a) Frequency response of an array of squares with wavy edges as shown in figure 5(a)

(b) Frequency response of an array of squares with wavy edges as shown in figure 5(b)

Figure 12: Frequency response of arrays of squares with wavy edges

5.7 Arrays of larger elements

The response shown in figure 13(a) clearly shows that fewer larger elements will give an uneven frequency response. Even here you can see what is probably the Reflection Filter effect, but the cut-off frequency can not be established, as the response has a very large dip right before the predicted value. The response shown in figure 13(b) is surprisingly smooth, but it does however have rather large dips from the mean value. This array also have a higher mean value than predicted. The Reflection Filter is prominent, and the cut-off frequency seems to be well predicted, although this array also have a peak near the predicted cut-off frequency.



(a) Frequency response of an array of squares as shown in figure 5(c)

(b) Frequency response of an array of rectangles as shown in figure 5(d)

Figure 13: Frequency response of arrays of larger elements

6 Conclusion and further work

The formula given in equation 2 seems to be a good estimate for the low limit frequency for the arrays measured in this report. However, one has to take in consideration that these measurements has tested a quite narrow range of this low limit frequency, but has found the prediction valid for several different geometries within the range tested. The measurements also showed a clear trend of a peak in the response near the predicted value of the cut-off frequency. The reason for this is not understood, but are worth further investigations.

The passband level predicted by the FK-filter seems to be a bit low. All the measurements done in this report had a mean value in the passband a few decibel higher than predicted. To get a smooth frequency response in the passband these measurements states that more small elements are preferred to fewer large ones, and they showed a trend that random patterns or complex periodic patters can potentially give a smoother response than periodic structured patterns. Even though well designed random patterns might give smoother response, they are more difficult to predict and to design. In an architectural point of view it would be easier to work with structured patterns.

A natural step further would be to investigate the behaviour of the Reflection Filter in other angles of incidence, and also test it over a larger range than in this report. One should also investigate the perceptual values of canopies. What frequency range is important in the reflection from canopies for the overall sound quality, and what are the effects on the overall sound quality of dips and peaks in the response of the reflection.

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